REINFORCED CONCRETE

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CROSBY LOCKWOOD & SON, LTD
20 TUDOR STREET, LONDON, E.C.4

First Published 1937
Second Edition 1944
(revised)
Reprinted 1945

MADE AND PRINTED IN GREAT BRITAIN

CONTENTS

Chapter *Page*

I MATERIALS USED IN REINFORCED CONCRETE - - - - - - - - 7
Cements; fine and coarse aggregates; water; steel.

II MIXING, PLACING AND TESTING - - 14
Concrete mixes; mixing, placing and curing; water-cement ratio; test cubes; concreting at low temperatures; consistency of concrete.

III PHYSICAL PROPERTIES: ELASTICITY: BASIC THEORY - - - - - - - 20
Physical properties of concrete and steel; modulus of elasticity and modular ratio; working stresses; cover and spacing of reinforcement; basic theory of bending.

IV DESIGN OF SIMPLE BEAMS AND SLABS 29
Design of simple beams and slabs; practical rules for beams and slabs; examples of design for beams and slabs.

V TEE BEAMS: ELL BEAMS: DOUBLE REINFORCEMENT - - - - - - - 36
British and American practice for T- and L-beams; examples of design; double reinforcement; steel beam theory; examples.

VI SHEAR STRESSES IN BEAMS - - - 54
Shear stresses; shear reinforcement; stirrups and diagonal bars; bond stresses and anchorage; practical considerations for detailing.

VII COLUMNS AND FOUNDATIONS - - 66
Types of columns and reinforcement; L.C.C. and other rules; economical design; foundations; safe pressure on sub-soils and on concrete; design and detail.

VIII REINFORCED CONCRETE CANTILEVER WALLS - - - - - - - - 80
Advantages and disadvantages of cantilever walls; earth pressures; types of R.C. walls; example of design; detailing; precautions against sliding.

IX PRESENT TENDENCIES IN R.C. DESIGN AND PRACTICE - - - - - - 88
Working stresses and modular ratio; R.C. frames; bending in columns; pre-cast units; pre-stressing; detailing; shuttering; surface treatment; conclusion.

INTRODUCTION
TO SECOND EDITION

In this edition the chapters dealing with the physical properties of concrete and its constituent materials have been revised and brought up to date. Chapter IX has been added and deals with recent developments in concrete practice, including such matters as surface treatment, detailing, etc. In the absence of the report of the Code of Practice Committee now engaged on this subject, stresses have been based mostly on the L.C.C. By-Laws and the American Society of Civil Engineers *Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete.*

The Author wishes to express his thanks to his colleague, Mr. R. H. Ray, for his help in revising the manuscript and trusts this small work will prove useful to those taking up the study of Reinforced Concrete.

SYMBOLS

FOR USE IN REINFORCED CONCRETE CALCULATIONS

A = area in general (in particular, area of concrete in compression).

A_b = equivalent area of spiral reinforcement (vol. of spiral per unit length of column).

A_c = area of steel in compression.

A_k = area of concrete in column core.

A_t = area of steel in tension.

A_w = area of web or shear reinforcement.

a = lever arm or distance between centroids of forces producing the resistance moment.

a_1 = lever arm ratio $= \frac{a}{d}$

B = overall breadth of concrete section.

b = Breadth or width of tee or ell in beam with slab.

b_r = Breadth or width of rib of beam.

C = total compressive force on section.

c = stress in concrete in compression.

D = diameter in general (in particular, of column head supporting flat slab, overall concrete depth; least lateral dimension of column).

d = Effective depth.

d_c = inset of compression steel.

d_s = Depth of slab or flange.

E_c = modulus of elasticity of concrete.

E_s = modulus of elasticity of steel.

I = moment of inertia.

K = coefficient; stiffness.

k = radius of gyration.

L = length in general (in particular, actual column height or centres of columns in flat slabs).

l = effective span of beam or slab or effective height of column.

M = bending moment.

M.R. = moment of resistance of section to bending.

m = modular ratio $= \frac{E_s}{E}$

n = neutral axis depth.

n_1 = neutral axis depth ratio $= n/d$.

O = sum of perimeters of reinforcement.

O.W. = Own weight,

P = pitch.

Q = coefficient in M.R. $= Qbd^2$.

R = reaction.

[continued on next page

SYMBOLS—*continued.*

r = steel ratio = $\frac{A_t}{bd}$ (tensile).

r_1 = steel ratio = $\frac{A_c}{bd}$ (compressive).

S = shear force.

s = shear stress.

s_b = bond stress.

T = total tension or tensile force on section.

t = tensile stress in reinforcement.

t_b = stress in spiral or helical binding reinforcement.

t_w = stress in web or shear reinforcement.

u = ultimate concrete stress = cube crushing strength.

W = total load on slab or beam.

w = load per unit area or length.

Z = section modulus.

$\left.\begin{array}{l} z_x = \\ z_y = \end{array}\right\}$ Bending moment coefficients for two-way slabs.

Chapter I

MATERIALS USED IN REINFORCED CONCRETE

Cements; fine and coarse aggregates; water; steel.

The materials used in reinforced concrete are:

(*a*) *Cement.* The cement used in modern practice may be of several kinds. The most commonly used is *Portland cement.* This is produced by burning calcareous (1) marls (2), or mixtures of clay and limestone till sintering (3) occurs and then grinding to a fine powder. *Rapid-hardening Portland cement* is produced by finer grinding. The ratio weight of lime: weight of silica, clay and iron oxide should be not less than 1·7. Both these cements are governed by British Standard Specification No. 12. *Portland blast-furnace cement* is produced by mixing Portland cement with finely ground slag which has been quenched. It contains more magnesium and sulphur than Portland cement. The specification governing this cement is British Standard Specification No. 146. *High Alumina* cement is produced from bauxite, shale, limestone and slag, which are first fused, not sintered, and then ground. It is a rapid hardening cement. There is a B.S. Specification No. 915 for this cement and the L.C.C. By-Laws specify tests as to fineness, chemical composition, setting-time and tensile strength of mortar composed of one part of cement and three parts of Leighton Buzzard sand.

Hydraulic admixtures include trass (4) which makes

(1) Of the nature of lime or limestone.
(2) Clay containing much calcareous matter.
(3) To form lumps.
(4) Rock containing pumice and other volcanic material used as a cement.

mortar or concrete more plastic and less permeable. It should be used with cements containing much lime.

Chemical Composition of Cement:

	Ordinary and Rapid-hardening Portland Cement	Portland Blast-furnace Cement	High Alumina Cement
CaO (lime)	59 - 66%	48 - 58%	35 - 40%
S_iO_2 (silica)... ...	18 - 23%	25 - 30%	10 - 12%
Al_2O_3 (alumina) ...	4.5 - 8%	10 - 20%	40 - 45%
Fe_2O_3 (iron oxide)	2.5 - 4.5%	3 - 5%	1 - 20%

The *hydraulic modulus* of cement is given by

$$\frac{\text{Percentage of CaO}}{\text{Percentages of } S_iO_2 + Al_2O_3 + Fe_2O_3}$$

the value for Portland cement being 1·7 to 2·4.

The *silica modulus* is given by

$$\frac{\text{Percentage of } S_iO_2}{\text{Percentages of } Al_2O_3 + Fe_2O_3}$$

the value varies between 1·2 and 4·0.

The hardening capacity of Portland cement increases with the *hydraulic modulus.* For greatest hardening capacity the *silica modulus* should be between 2·5 and 3·0.

Cements with a high silica (S_iO_2) content or low alumina (Al_2O_3) content set and harden slowly. Cements with high alumina content are quick setting and rapid hardening. Gypsum may be added in small quantities to control setting time. SO_3 (sulphur trioxide) content not to be more than 2·5%.

During setting a rise of temperature takes place in the first hours and afterwards a gradual fall in temperature takes place. This is most marked in the case of high alumina

cement. Low-heat cements may be used in certain cases. They contain tricalcium aluminate and silicate. Special cements may be used for particular purposes. (For more detailed information on this point see paper by Dr. Lea before the Institution of Civil Engineers on "Modern Developments in Cement in Relation to Concrete Practice" on 15th December, 1942).

STANDARD TESTS FOR CEMENT (*see B.S. Specifications Nos.* 12, 146 and 915).

(1) *Fineness.* This is found by means of a sieve analysis and the percentage retained on B.S. Test Sieve No. 170 should not exceed 10 for Ordinary and 5 for Rapid-hardening Portland cement.

(2) *Chemical Composition and Loss of Weight on Ignition.*
(*c*) *Strength.* Tests for tensile and compressive strength are made on briquettes composed of cement and sand.
(*d*) *Setting Time.* This test is usually carried out by means of the Vicat apparatus and both the initial and final setting times should be measured.
(*e*) *Soundness.* Test is carried out by "Le Chatelier" method, and test specimen is boiled for three hours and expansion measured.

USES OF VARIOUS CEMENTS.

Rapid hardening or high alumina cements are used to save time. Slow-setting and hardening Portland cement is used for dams, weirs, and large masses of concrete. High alumina cement is expensive and requires more water on account of its high setting temperatures.

(*b*) *Fine Aggregate.* This is generally taken as aggregate passing a $\frac{3}{16}''$ mesh. The necessary properties are hardness, cleanliness, and freedom from such impurities as clay or organic matter. The shape of the grains is not very important

and rounded grains may be as useful as sharp-edged grains. Impurities can be detected by shaking the aggregate with water in a vessel and noting the amount of matter in suspension. Wherever impurities occur, they should be removed by washing. River or fresh water sand should not require washing, but pit sands usually require this treatment. All such sands can be used for reinforced concrete work. Sea sand may be used if not too fine and if taken from below high water mark. If taken from above the high water mark it may contain too much salt and in doubtful cases it is as well to make test cubes. Foundry and silver sands are usually too fine. Screenings from crushed stone may be used if they do not contain too much dust. Limestone or whinstone grit should not be used. Crushed bricks are generally not suitable. Aggregates should not contain soluble salts. Crushed hydraulic slags may be used. Silt, clay and loam hinder the bond. The percentage of these materials can be found by shaking with water in a vessel and pouring off the water until the wash water is clean and finding the percentage loss of weight. The size of the fine aggregate varies with the purpose for which it is intended. The grading of the fine aggregate is important, and this is generally done by means of a sieve analysis. Not more than 5 per cent of the weight should pass a No. 100 sieve. The aggregate should be graded between this limit and the upper limit (usually $\frac{3}{16}''$) so as to obtain as dense a concrete as possible. The ideal sieve analysis curve is called the Fuller curve. Actually the grading of the fine aggregate has a greater effect on the strength of the concrete than the grading of the coarse aggregate. The sizes of sieves used for the purpose of grading fine aggregate are laid down in British Standard Specification for Test Sieves No. 410. Some sands are naturally well graded but others may require to be mixed in order to obtain concrete of a high strength or high water resisting properties.

(c) *Coarse Aggregate.* This should possess the following qualities: hardness, freedom from impurities such as clay, loam, organic matter, etc.; shape, cubical or spherical, freedom from certain chemicals and in certain cases have high fire-resisting properties. It is generally gravel or crushed natural stone. It should be retained on a $\frac{3}{16}''$ mesh and the upper limit should be $\frac{1}{4}''$ less than the least lateral space between reinforcing bars. The maximum size varies with the purpose for which the concrete is to be used.

Maximum Sizes of Coarse Aggregate for Various Purposes.

Reinforced foundation walls and footings ...	$1\frac{1}{2}''$
Slabs, beams and reinforced walls	$1''$
Columns	$1''$

Coarse aggregate should be obtained from a source which has proved satisfactory over a period of years. Hard brick or tile may be used for mass concrete work. Aggregates containing a large amount of impurities, which must be removed by washing, are not economical in use. Dust must be removed by screening. The hardness of aggregate for reinforced concrete should be such as to give a crushing strength of 5,000 lbs./inch^2. Softer aggregates can be used for mass concrete work. Aggregates composed of broken bricks are porous and should not be used for reinforced concrete. The amount of water absorbed in 24 hours' immersion should not exceed 5 per cent of weight for reinforced concrete and 10 per cent of weight for mass concrete. Gravel from pit and river, and Thames ballast, are most commonly used and are quite satisfactory after being washed to remove impurities. Pit gravel generally requires more washing than river gravel. A natural gravel has fewer voids than crushed stone and therefore is better where water-resisting concrete is required. Gravel found during excavation for foundations, etc., should not be used without washing and screening. Beach shingle, if used,

should be taken from below high water mark. The best natural stones for concrete are granite, whinstone, quartzite, basalt flint and some of the harder sandstones. Porous sandstones should not be used. Granite produces good concrete with an excellent wearing surface. Flint and Portland stone are satisfactory aggregates but flint and quartzite should not be used where fire-resisting properties are required. Whinstone is a satisfactory aggregate. Broken concrete may be used for mass work but not for reinforced concrete. Shale or similar materials are not suitable, but the harder limestones are suitable after screening. L.C.C. By-laws ban the use of broken brick in reinforced concrete, although it has a good resistance to fire and is lighter than gravel concrete. Coal residues (including clinker, ashes, coke-breeze, pan-breeze, slag, etc.), copper slag, forge breeze, dross, etc., must not be used as they contain chemicals which would attack the reinforcement. Good blast furnace slag is a suitable aggregate. Foamed blast furnace slag is a light weight aggregate and is specified by B.S. Spec. No. 167. Blast furnace slag contains about 30 per cent silicates, 10 per cent alumina, 40 per cent lime, etc. Sulphides should not be present. Some of the aggregates which are not suitable for R.C. work may be used in mass concrete, partitions, etc. Pumice produces a light concrete, but great care must be used in grading. It is useful for partitions and fire protection. The grading of coarse aggregate does not play so great a part as that of the fire aggregate in determining the strength of concrete. The usual practice in this country is $\frac{3}{16}''$ to $\frac{3}{4}''$ for R.C. work, the percentage less than $\frac{3}{16}''$ being not more than 10 and the percentage less than $\frac{3}{8}''$ not more than 25.

(*d*) *Water.* Water used for concrete should be clean, fresh, and free from oils, acids. In general, water that is suitable for drinking is suitable for concrete. In doubtful cases a mortar strength test should be made. Soft water

gives a lower strength than hard water. Water containing gas or free acid is injurious, also water from swamps and marshes. Water containing salts should not be used with high alumina cement. The presence of acids can be detected by litmus paper. Sulphates can be found by acidifying the water with dilute hydrochloric acid and adding a dilute solution of barium chloride. The formation of a white precipitate denotes the presence of sulphates. The amount of water required varies and this will be discussed under "Water-Cement Ratio."

(*e*) *Steel.* Steel used as reinforcement may be in the form of bars (plain or deformed), wire mesh, expanded metal, etc. It should be free from loose dust, mill scale, oil or other substances which tend to destroy the bond between steel and concrete. Mild steel used must be in accordance with British Standard Specification No. 15 (Steel for Bridge and General Building Construction). L.C.C. By-laws allow the use of steel complying with B.S. Specification 165 in slabs only. *High tensile steel should be in accordance with B.S. Specification 548 (H.T. Steel for Bridges, etc.), but this steel is expensive and should be used only in conjunction with high grade concrete. Mesh reinforcement for concrete work should be in accordance with B.S. Specification No. 785 (Hard-drawn Steel Wire). The cost per ton of this type of reinforcement is considerably higher than that of mild steel rods but the cost of bending and placing is lower. Particulars of such mesh can be obtained from the makers' catalogues. Another form of reinforcement sometimes used is expanded metal, which is made by slotting a flat steel plate and pulling it out into a form of grid or mesh. Rods used for reinforcement vary in diameter from $\frac{1}{4}''$ min. to $2''$ for main steel, but $\frac{3}{16}''$ diameter is permitted for use as hoops, stirrups, etc. The minimum diameter of wire allowed for mesh reinforcement under the L.C.C. By-laws is $\frac{1}{10}''$.

* Incorporated in B.S.S. 785.

Chapter II

MIXING, PLACING AND TESTING

Concrete Mixes; Mixing, Placing and Curing; Water-cement Ratio; Test Cubes; Concreting at Low Temperatures; Consistency of Concrete.

CONCRETE MIXES. These vary according to the purpose for which the concrete is to be used. Mixes are by volume and not by weight. As cement weighs 90 lbs. per cubic foot, a cwt. bag of cement contains $1\frac{1}{4}$ cub. ft. Coarse and fine aggregate are measured separately in gauging boxes made to suit the particular mix. The most common mix for R.C. work is 1 : 2 : 4 ; *i.e.* $2\frac{1}{2}$ cub. ft. of fine and 5 cub. ft. of coarse aggregate per cwt. bag of cement. This mix is suitable for most kinds of R.C. members:- beams, columns, slabs, panel walls, etc. A 1 : $1\frac{1}{2}$: 3 mix contains $1\frac{7}{8}$ cub. ft. of fine and $3\frac{3}{4}$ cub. ft. of coarse aggregate per cwt. bag of cement. Similarly, a 1 : 1 : 2 contains $1\frac{1}{4}$ cub. ft. of fine and $2\frac{1}{2}$ cub. ft. of coarse aggregate per cwt. bag of cement. Another mix which may be used is 1 : 1·2 : 2·4. With regard to concrete mixes it is interesting to note that the American Society of Civil Engineers Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete (June, 1940) lay down two alternatives: (1) the proportions for cement, aggregate and water are specified, (2) in which the quality of the concrete (as regards strength and exposure) is specified and the contractor is free to vary the mix and water-cement ratio within certain limits.

The mixes included in the range 1 : 1 : 2 to 1 : 2 : 4 are for high class and reinforced concrete. For mass concrete leaner mixes can be used, *e.g.* 1 : 3 : 6 for foundations, retaining walls, etc. This has $3\frac{3}{4}$ cub. ft. of fine and $7\frac{1}{2}$ cub. ft. of coarse aggregate per cwt. bag of cement. Mixes

for mass concrete are often stated as cement: total aggregate, *i.e.* 1:6; 1:8; 1:10 and 1:12, *i.e.* 1 cwt. bag to 9, 12, 15 and 18 cub. ft. of total aggregate. The amount of materials required to produce 1 cub. yd. of concrete of various mixes is given in Table I.

MIXING. Wherever possible mixing of concrete should be carried out by a mechanical or batch mixer, preferably one in which the amount of water used is easily controllable. In cases where hand mixing is unavoidable, the mixture should be turned over twice dry and twice wet. Another kind of concrete which has been introduced in recent years is "ready-mixed". This may be mixed wholly or partially at a central plant and a period for delivery varying from 1½ hours to 2 or 3 hours must be adhered to (depending on the nature of the aggregate, the mix and the air temperature). The minimum period for delivery is probably about one hour.

PLACING. Concrete should be deposited without segregation of the various parts. This can be done by hand, by vibration, by pumping, or by pneumatic methods. In this country hand placing of concrete is the most common method. Care must be taken to work the concrete round the steel by rodding or punning. Concrete can also be placed by vibrators. This method must be used carefully as it is comparatively new. It allows a certain amount of economy as stiffer mixes with lower water content can be used. Where excessive bleeding (*i.e.* escape of water) occurs, the amount of fine sand should be increased. The appearance of a line of cement paste at the junction of concrete and form, or concrete and steel, is evidence that the concrete is sufficiently vibrated. Vibration increases the pressure on the formwork, which should be made correspondingly

stronger. Vibration should produce denser and stronger concrete.

Pneumatic placing of concrete is used more in America than in this country. The concrete used should be of medium consistency and not more than 7 cub. ft. discharged at a time, and the distance from "gun" to the nozzle should not be more than 1,000 feet.

Concrete can be placed by pumping, the pressure being not less than 300 lbs./in^2. The pipe line should have as few bends as possible. The "slump" or consistency of concrete will vary with the diameter of the pipe and the nature of the aggregate.

CURING. During setting a rise of temperature takes place, this being most marked in the case of high alumina cement. For this reason, and also to prevent loss of moisture with consequent fall of strength, it is necessary to "cure" the concrete. This can be done by covering the "green" concrete with damp sacks or sand or by sprinkling with water. In the case of high alumina cement, this should be done within 6-7 hours of mixing and continued till the concrete is 24 hours old (curing after 24 hours has elapsed is very little use). The period for curing Portland cement concrete varies from 3 to 7 days.

WATER-CEMENT RATIO. This ratio $\left(\frac{\text{weight of water}}{\text{weight of cement}}\right)$ has a considerable influence on the strength of concrete. Finely-ground cements (*e.g.* rapid-hardening Portland cement) with high setting temperatures, require more water than coarser ground cements with low setting temperatures. Concretes whose aggregates contain a relatively large proportion of sand require more water to obtain a given consistence than those with a coarser aggregate. Broken stone aggregate requires more water than natural gravel and sand.

Tests show that a water-cement ratio of 0·4 for 1 : 2 : 4 concrete gives maximum strength and that strength falls if more water is added. The corresponding figure for 1 : 1½ : 2½ concrete is 0·35. In important work it is as well to specify a *maximum* water-cement ratio as a usual fault among workmen is to use too much water in order to make the concrete flow round the steel. The water-cement ratio will vary with the materials and purpose for which the concrete is to be used. It is usually expressed in gallons of water per cwt. bag of cement. (1 gallon of water weighs 10 lbs., so for 0·4 w./c. ratio 4½ gallons are required per bag of cement).

TEST CUBES. Test cubes of concrete are made for two kinds of test: (*a*) preliminary, and (*b*) works.

(*a*) In this case the test cube (6″ edge) is made in the laboratory from materials such as are to be used on site and should be mixed in a small batch mixer or by hand. The cement and fine aggregate should first be mixed dry, and coarse aggregate added and mixed. Water should be added and the whole mixed for not less than 2 minutes. The concrete should then be moulded by placing the concrete in the mould in three layers, each layer being rammed by a steel rammer weighing 4 lbs. and having a face 1″ square and 15″ long. Afterwards, the specimens should be stored in moist air at 58 - 64°F. for 24 hours and then in water at same temperature until they are tested (at 28 days). They are tested between steel plates, the rate of loading being approximately 2,000 lbs./in.^{2} per minute.

(*b*) Works tests are somewhat similar to preliminary tests except that specimens should be taken from the concrete used on the site immediately after placing. They should be stored at the site for 24 hours under damp sacks (in the moulds) and afterwards buried in damp sand, until they

are required for testing, at a temperature of not less than 40°F.

In both preliminary and works tests, readings should be taken of the slump of the concrete. Test cubes are most important in controlling the strength of concrete and form the most logical basis for assessing the working stresses in concrete.

CONCRETING AT LOW TEMPERATURES. Concreting at temperatures near to or below freezing point (32°F.) is most undesirable, but in certain cases it may be unavoidable, where the work is of an urgent nature. In such cases the materials used for concreting may be treated, or alternatively, 2 lbs. of calcium chloride may be added per cwt. bag of cement. This has generally a beneficial effect on the strength of the concrete.

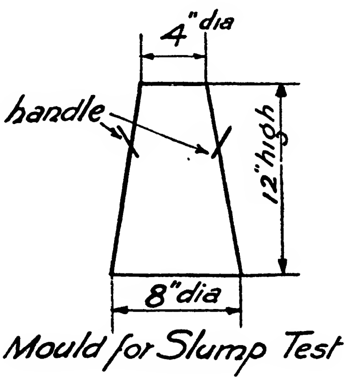


FIG. 1

CONSISTENCY OF CONCRETE. The consistency or workability is a most important factor in concrete. It is generally

controlled by means of a slump test. This is carried out by filling a mould 4″ diam. at top and 8″ diam. at base and 12″ high with concrete and noting the slump of the concrete when the mould is removed. The slump for R.C. work should never exceed 6″. Maximum slumps are—beams, slabs, walls, etc., 6″; walls 4″; foundations 3″. For mass concrete the maximum slump is 2″ for foundations and 5″ for walls and slabs (*see* Fig. 1).

The American Society of Civil Engineers give a number of maximum and minimum slumps for a large variety of concrete work.

Consistency can also be tested by a flow test or flow table (table is shaken and the ratio diameter of base of concrete : diameter of base of mould, gives the measure of consistency) or by a vibrating table.

"Dry" concrete becomes moist on the surface after ramming and can be placed only by ramming. It contains just enough water to be formed into a ball in the hands. It can be used for mass work, foundations, etc.

"Plastic", wet or soft concrete contains rather more water.

"Poured" concrete is placed by chutes and contains an excess of water.

"Wet" concrete is concrete which flows round the reinforcement easily and should consist of well graded aggregates.

The workability of concrete depends on a number of factors and must be varied to suit the purpose for which the concrete is to be used.

"Bleeding" is the escape of water from freshly-placed concrete, commonly noted as an accumulation upon a horizontal surface.

"Laitance" is extremely fine material of little or no hardness which may collect on the surface of freshly-placed concrete or mortar, resulting from excess mixing water.

"Honeycomb" is a surface or interior defect in a concrete mass characterized by the lack of mortar between the coarse aggregate particles.

Chapter III

PHYSICAL PROPERTIES : ELASTICITY: BASIC THEORY

Physical Properties of Concrete and Steel; Modulus of Elasticity and Modular Ratio; Working Stresses; Cover and Spacing of Reinforcement; Basic Theory of Bending.

CONCRETE is strong in compression but weak in tension and it is usual to neglect its tensile strength and use steel to take up the tension. When considering the compressive strength of concrete, it should be borne in mind that its ultimate strength is probably much higher than that indicated by a test cube. This is due to the hardening of concrete (which is quite distinct from setting). This process is gradual and the concrete does not attain its maximum strength for some considerable time. The increase of strength is rapid for the first 28 days, and then the rate of increase is much slower, the rate depending on the materials. The most reliable evidence of the compressive strength is, of course, the preliminary test on the 6″ cube, and in this country it is usual to take one-third of this value as the safe bending compressive stress, thus

$$c = u/3$$

This value c might be called the "basic" stress as other safe stresses in the concrete are taken in proportion to it. Note that the direct compressive stress is generally taken as 80 per cent of c. The shear stress s is taken as $\frac{1}{10}$ of c,

with the proviso that "punching" shear stress may be taken as $\frac{2}{10}$ of c. The bond stress s_b is generally taken at 13 to 14 per cent of c, although recent exhaustive tests have shown that s_b does not rise quite in proportion to c, and in higher grade concretes, it is as well to be conservative in fixing bond stresses.

American practice is somewhat different from British in this respect. The most recent regulations allow $c=45$ per cent of u; direct compression $= 25$ per cent of u; shear (s) from 2 to 6 per cent of u and $s_b = 3$ to 5 per cent of u.

The modulus of elasticity of concrete (E_c) is an important factor in R.C. calculations. Actually, concrete has no clearly defined elastic limit as during tests it is difficult to distinguish between plastic and elastic deformations. Some authorities state a constant figure of $E_c = 2 \times 10^6$ lbs./in.^{2}, but actually E_c varies with the composition and strength of the concrete. Some authorities take a varying value so that as u increases, E_c increases.

STEEL, on the other hand, is a homogeneous material having a clearly defined elastic limit and a modulus of elasticity E_s, which is more or less the same for high tensile as for mild steel, the value being 30×10^6 lbs./in.^{2} Mild steel to B.S. Specification No. 15 has an ultimate strength in tension of 28-33 tons/in.^{2}, with a yield point of about 18 tons/in.^{2} The yield point is important as many failures of R.C. Beams are due to yielding of the reinforcement. The ultimate strength for H.T. steel to B.S. Specification 548 is 37-43 tons/in.^{2}, with a correspondingly higher yield point. The permissible tensile stress in M.S. rods used in R.C. work is usually 18,000 to 20,000 lbs./in.^{2}, and for H.T. steel and hard-drawn steel wire from 25,000 to 27,000 lbs./in.^{2}

MODULAR RATIO. $m = \frac{E_s}{E_c}$

As E_s is constant for steel, it will be seen that m varies with E_c. Some authorities have laid down that $m = \frac{40,000}{u}$, which means that m decreases as u increases. L.C.C. By-laws adopt a constant value of 15 for m, but American regulations give the values as below:—

u lbs./in.^{2}			m
2,000 to 2,400	...	...	15
2,500 to 2,900	...	...	12
3,000 to 3,900	...	...	10
4,000 to 4,900	...	...	8
over 5,000	...	...	6

This agrees fairly well with the formula $m = \frac{40,000}{u}$ and appears to be a logical rule as well as agreeing with experimental values.

REINFORCEMENT. In order to take up tensile forces and shear, the steel used as reinforcement must be placed so that the R.C. member is able to develop its maximum strength. The concrete cover of reinforcement should never

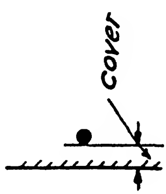


FIG. 2

be less than that laid down in the building regulations. For thin walls and slabs the minimum cover is ½″, while for beams, columns, etc., it should be not less than 1″.

Wherever there is a likelihood of corrosion due to fumes or other deleterious matters, these values should be increased. For concrete foundations, the cover should be not less than 3″, and for other members subject to severe weather conditions 2″ should be the minimum cover (*see* Fig. 2).

The spacing of reinforcement is important. The lateral spacing of bars should allow a clear distance equal to the diameter (or diameter of larger bar) or ¼″ more than the maximum size of coarse aggregate used, whichever is the greater. The vertical spacing of bars may be not less than ½″ except where bars cross at right angles, but it should be borne in mind that splices may have to be allowed for in fixing the spacing of steel. Where bars intersect at right angles they should be tied together with No. 16 S.W.G. soft iron wire or clips. American regulations insist on wider spacing than the L.C.C. By-laws.

WORKING STRESSES. It is rather difficult to lay down definite figures for these as the whole subject is being examined by a Code of Practice Committee composed of members nominated by the various engineering institutions and other interested bodies. Comparative values are given in Table II in the Appendix.

BASIC THEORY OF BENDING. This is based on these assumptions:—

(1) Plane sections normal to the axis remain plane after bending.

(2) Tensile strength of concrete is neglected.

(3) Stress is proportional to strain.

(4) The modular ratio m for any given concrete is constant within the range of stress due to the loading.

(5) Adhesion (or bond) between the steel and concrete is perfect within the limit of proportionality (elastic limit) of the steel.

This is the generally accepted theory, although in recent years certain theories neglecting the effect of the modular ratio have been put forward. The above method certainly errs on the side of safety, if at all, and the student or beginner would do well to familiarise himself with it before entering upon the more controversial methods.

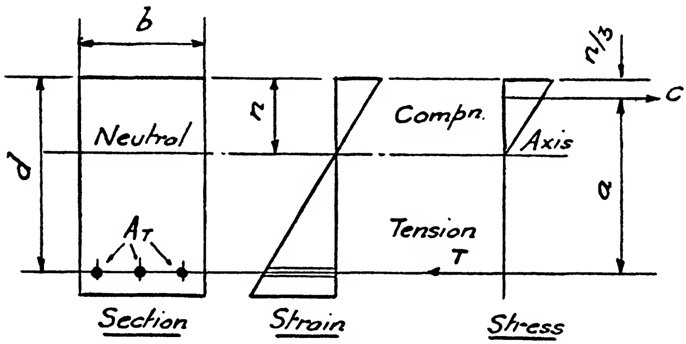


FIG. 3.

The basic theory is sometimes called the straight-line no-tension method. Referring to Figure 3,

since $E_s = \dfrac{\text{stress in steel}}{\text{strain in steel}}$, stress in steel $= E_s \times$ strain in steel.

Similarly, $E_c = \dfrac{\text{stress in concrete}}{\text{strain in concrete}}$, stress in concrete $=$

$$E_c \times \text{strain in concrete, and } m = \frac{E_s}{E_c}.$$

$$\therefore \frac{\text{Stress in steel}}{\text{Stress in concrete}} = \frac{\text{strain in steel}}{\text{strain in concrete}} \times \frac{E_s}{E_c}$$

$$= m \times \frac{\text{strain in steel}}{\text{strain in concrete}}.$$

But strain in steel = strain in concrete at same point, *i.e.* at same distance from neutral axis.

Also to produce the same strain in steel as in concrete, it is necessary to apply m times the stress on the concrete.

By the standard theory of bending

$\frac{M}{I} = \frac{f}{y}$, *i.e.* stress is proportional to distance from neutral axis.

$$\frac{\text{Stress in concrete at top}}{\text{Stress in concrete at } d \text{ from top}} = \frac{n}{d-n},$$

but stress in steel = m × stress in concrete (at same point)

$$\therefore \frac{c}{\text{stress in concrete at } d \text{ from top}} = \frac{n}{d-n}.$$

$$\frac{c}{t/m} = \frac{n}{d-n}$$

$$\therefore \quad t = c\left(\frac{d-n}{n}\right) \times m \quad . \quad . \quad . \quad . \quad (1)$$

If A_t is area of tensile reinforcement.

Total tensile force $T = A_t \times c\left(\frac{d-n}{n}\right) \times m$.

The compressive stress varies from zero at the neutral axis to c at the top, the average stress being $c/2$.

$\therefore$ The total compressive force $C = \frac{c}{2} \times bn$

$$= \frac{bcn}{2}$$

For equilibrium $C = T$

$$\therefore A_t \times c\left(\frac{d-n}{n}\right) \times m = \frac{bcn}{2} \quad . \quad . \quad . \quad (2)$$

$$or\ A_t \times m\left(\frac{d-n}{n}\right) = \frac{bn}{2}$$

The force C acts at the centre of gravity of the triangle representing the compressive stress, *i.e.* at $\left(n - \frac{n}{3}\right) = \frac{2}{3}n$ from the neutral axis. The forces *C* and *T* being equal, form a couple called the Moment of Resistance (M.R.) of the section which must equal the applied bending moment (M).

Taking the compression force *C*, the moment of resistance

$$M.R._c = \frac{bcn}{2} \times \left(d - \frac{n}{3}\right) \quad . \quad . \quad . \quad (3)$$

In the same way

$$M.R._t = A_t \times t \times \left(d - \frac{n}{3}\right) \quad . \quad . \quad . \quad (4)$$

The factor $\left(d - \frac{n}{3}\right)$ is the lever arm *a* of the section and varies according to *t*, *c* and *m*.

Writing $a = a_1 d$
and $n = n_1 d$
also $r = A_t/bd$ = steel ratio
$\therefore A_t = rbd$

Equation (3) becomes

$$M.R._c = \frac{bcn_1 d}{2} \times \left(d - \frac{n_1 d}{3}\right)$$

$$= \frac{bcn_1 d}{2} \times a_1 d$$

$$= bd^2 \times \frac{cn_1 a_1}{2}$$

$$= bd^2 \times \frac{cn_1\left(1 - \frac{n_1}{3}\right)}{2}$$

$$= Qbd^2$$

$$\text{where } Q = \frac{cn_1\left(1 - \frac{n_1}{3}\right)}{2} = \frac{cn_1 a_1}{2}$$

For any given values of t, c and m, the values of n, a and Q can be found and tabulated. In the same way, the value of r can be found. This is the steel ratio which should be used if the steel and concrete are to be stressed to their permissible limits at the same time.

$$\text{Writing } T = A_t \times t$$
$$= rbd \times t$$

$$\text{and } C = \frac{bcn}{2}$$

$$\text{but } C = T$$

$$\therefore \frac{bcn}{2} = rbd \times t$$

$$\frac{bcn_1 d}{2} = rbdt$$

$$\therefore r = \frac{cn_1}{2t}$$

The values of n_1, a_1, Q and r for various values of t, c and m are given in Tables IIA and IIB.

Take for example the rather conservative values

$$t = 18{,}000 \text{ lbs./in.}^2$$
$$c = 750 \text{ lbs./in.}^2$$
$$\text{and } m = 15.$$

Then from equation (1)

$$18{,}000 = 750\left(\frac{d - n}{n}\right) \times 15$$

$$\therefore\ 24 = 15\left(\frac{d}{n} - 1\right)$$

$$\therefore\ 1\cdot6 = \frac{d}{n} - 1 \quad or \quad n_1 = \frac{1}{2\cdot6} = \cdot385$$

$$a_1 = 1 - \frac{\cdot385}{3} = \cdot872$$

$$Q = \frac{750}{2} \times \cdot385 \times \cdot872 = 125\cdot7$$

$$r = \frac{750 \times \cdot385}{2 \times 18{,}000} = \cdot008$$

If t is increased to 20,000 lbs./in.^{2}, $n_1 = \cdot36$, $a_1 = \cdot88$, $Q = 119$, $r = \cdot00675$, so that the higher value of t does not result in increased values of Q. On studying the values given in the table it will be seen that Q increases with the richness of the mix, *i.e.* as c increases Q increases accordingly. Higher values of t do not increase the value of Q unless the value of c also rises, in other words, it is not economical to use high tensile steel unless in conjunction with high-grade concrete. The ratio Q/r rises with t, and therefore there is a certain economy in steel and, of course, steel is costly as compared with concrete. In this connection it is as well to examine the effects of using hard-drawn steel wire stressed to 25,000 to 27,000 lbs./in.^{2}. Comparing the values for this reinforcement with those for mild steel reinforcement, it will be seen that the values for Q in the case of hard drawn steel wire are actually *less* than those for M.S. reinforcement. The steel ratio r is lower, but bearing in mind the much higher price of this type of reinforcement, it is doubtful whether any economy is

effected. The chief advantage of this reinforcement is that it can be used by unskilled labour or in small jobs in conjunction with steelwork or in cases where speed of construction is essential.

Chapter IV

DESIGN OF SIMPLE BEAMS AND SLABS

Design of simple Beams and Slabs;
Rules for Beams and Slabs.

HAVING established the relations between t, c, m and other constants for concrete beams, we can now apply them to the design of beams and slabs.

SLABS. Slabs may be supported on two or four edges and they are generally supported on beams with which they are monolithic. Slabs may also form a "mushroom" type of floor in which the slabs are supported on columns directly. In most concrete construction (apart from pre-cast concrete) the slabs, beams and columns are cast monolithically and the design should allow for continuity and corresponding transference of bending moments from one member to another. At the same time, the designer should understand the design of the individual members before proceeding to the design of complete buildings, etc.

In general, it can be said that thin slabs are relatively more expensive than thick slabs and some regulations give 4″ as the minimum thickness. Slabs may be reinforced in one (one-way slabs) or two directions at right angles (two-way slabs). For one-way slabs, secondary or distribution steel should be placed at right angles to the main reinforcement at a spacing not exceeding four times the effective depth of the slab. The total area of the secondary

steel should be not less than 10 per cent of that of the main steel. The effective depth (to centre of steel) should be not less than span /20 for one-way and *span /30 for two-way slabs.*

Example 1.—Design a singly reinforced slab to carry a superimposed load of 2 cwts. per square foot over a single span of 8 feet. Take $c=750$ lbs./in.^{2}; $t=18{,}000$ lbs./in.^{2} and $m=18$. For a span of 8 ft. the minimum effective depth is $96/20=4{\cdot}8''$; assume overall depth 6″ and weight of concrete 144 lbs. per cub. ft., *i.e.* 12 lbs./ft.^{2} per inch thickness.

Superimposed load	$\doteq$ 224 lbs./ft.^{2}	
Dead load $= 6\times 12$	= 72	,,
Total	= 296	,,

$$\text{Max. B.M. per foot width} = \frac{296\times 8^2\times 12}{8}$$
$$= 28{,}416 \text{ in lbs.}$$

This must equal the M.R. of the slab.

For given values:—$n_1=0{\cdot}43$; $a_1=0{\cdot}86$; $Q=138$; $r={\cdot}009$

$$\therefore\ 28{,}416 = 138\times 12\times d^2$$
$$d^2 = 17{\cdot}1 \qquad d=4{\cdot}12''$$
$$\text{As } a=0{\cdot}86\times 4{\cdot}12 = 3{\cdot}55$$

$$A_t = \frac{28{,}416}{18{,}000\times 3{\cdot}55} = {\cdot}45 \text{ in.}^2 \text{ per ft. width.}$$

Referring to Table IV, we find that ½″ bars at 5″ centres give an area of ·47 in.^{2} per ft. width. Overall depth of slab becomes $4{\cdot}8+{\cdot}25+{\cdot}50=5{\cdot}55''$. The overall depth can be made 6″ (*see* Fig. 4).

For secondary reinforcement the area per ft. width must

not be less than ·045 in.² and the spacing not greater than $4 \times 4 \cdot 12 = 16 \cdot 5$ in.

$\frac{1}{4}''$ bars at 12″ centres give an area of ·049 in.² per ft. width and can be adopted.

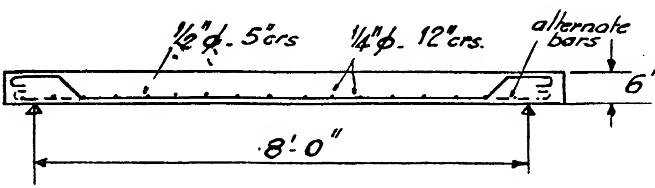


FIG. 4.

Two-way Slabs. When a slab is reinforced in two directions at right angles, the shorter span being stiffer carries a greater proportion of the load than the longer span. Various authorities give different values for the distribution.

If $K = \dfrac{\text{long span}}{\text{short span}}$

z_x = proportion of load on short span.

z_y = „ „ „ long span.

Grashof and Rankine formula is

$$z_x = \frac{K^4}{K^4+1}$$

$$z_y = 1 - \frac{K^4}{K^4+1}$$

French Government Formula is

$$z_x = \frac{K^4}{K^4+2}$$

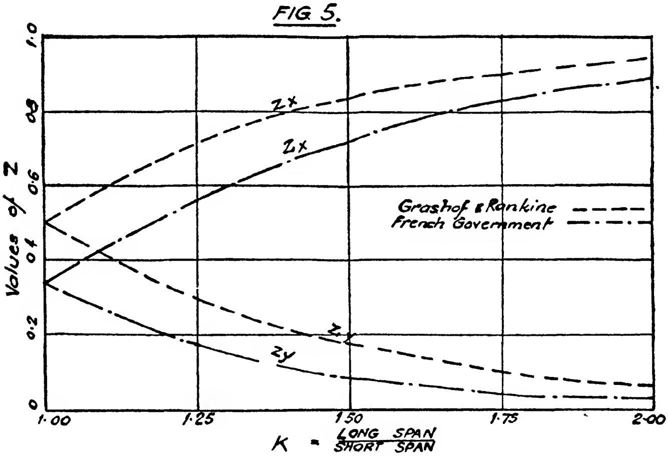
$$z_y = \frac{1}{1+2K^4}$$

Coefficients for one-way and two-way slabs are given by the American Society of Civil Engineers and by the Code of Practice of the D.S.I.R.

K	Grashof and Rankine z_x	Grashof and Rankine z_y	French Government z_x	French Government z_y
1·00	0·50	0·50	0·33	0·33
1·25	0·71	0·29	0·55	0·17
1·50	0·83	0·17	0·71	0·09
1·75	0·90	0·10	0·83	0·05
2·00	0·94	0·06	0·89	0·03

(*See* Fig. 5).

FIG. 5.



Example 2.—Design a slab 10 ft. long and 8 ft. wide supported on four edges and carrying a load of 300 lbs. per square foot. The effective depth is not less than 96/30=3·2″. Take overall depth=4″.

Using $m = 18$; c=750 lbs./in.² and t=18,000 lbs./in.²

$$K = \frac{10}{8} = 1{\cdot}25$$

Using Grashof and Rankine formula

$z_x = 0{\cdot}71$; $z_y = 0{\cdot}29$

Load on shorter span $= 0{\cdot}71 \times$ total load.

" " longer " $= 0{\cdot}29 \times$ " "

	lbs./ft.^{2}
Superimposed load	$= 300$
Self weight$=4 \times 12$	$= 48$
Total	$= 348$

Total load $= 348 \times 8 \times 10 = 27{,}840$ lbs.

Load on shorter span $= 0{\cdot}71 \times 27{,}840 = 19{,}750$ lbs.

" " longer " $= 0{\cdot}29 \times 27{,}840 = 8{,}090$ "

B.M. on shorter span $= \dfrac{19{,}750 \times 8 \times 12}{8}$ in lbs.

B.M. on shorter span (per ft. width) $= \dfrac{19{,}750 \times 8 \times 12}{8 \times 10}$

$= 23{,}700$ in lbs.

$$\therefore\ d^2 = \frac{23{,}700}{12 \times 138} = 14{\cdot}3 \quad d = 3{\cdot}75''$$

$A_t = 3{\cdot}75 \times 12 \times {\cdot}009 = {\cdot}405$ in.^{2}

$\frac{1}{2}''$ bars at $5\frac{1}{2}''$ centres give an area $= {\cdot}428$ in.^{2}

Overall depth $= 3{\cdot}75 + {\cdot}25 + 0{\cdot}50 = 4\frac{1}{2}''$

B.M. on longer span $= \dfrac{8{,}090 \times 10 \times 12}{8}$ in lbs.

" " " (per ft. width) $= \dfrac{8{,}090 \times 10 \times 12}{8 \times 8} = 15{,}200$ in lbs.

$$A_t = \frac{15{,}2000}{18{,}000 \times {\cdot}86 \times 3{\cdot}75} = 0{\cdot}26 \text{ in.}^2$$

$\frac{1}{2}''$ diam. bars at $9''$ centres give an area of $\cdot 262$ in.^{2}

(*See* Fig. 6)

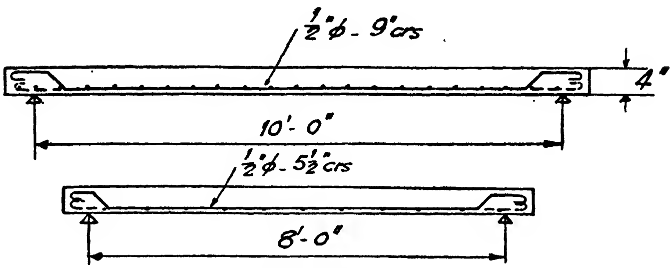


FIG. 6.

Example 3.—Design a beam $10''$ wide carrying a total B.M. of 250,000 in lbs. with values as before. Find overall depth of beam and reinforcement required.

$$
\begin{aligned}
\text{Let effective depth} &= d'' \\
\text{Then M.R.} &= 138 \times 10d^2 \\
&= 1380d^2 \\
&= 250{,}000 \text{ in lbs.} \\
d^2 &= \frac{250{,}000}{138} = 181 \text{ approx.} \\
d &= 13{\cdot}45''
\end{aligned}
$$

Then

$$
\begin{aligned}
A_t &= 10 \times 13{\cdot}45 \times {\cdot}009 = 1{\cdot}21 \text{ in.}^2 \\
\text{Area of } 2 - \tfrac{7}{8}'' \text{ diam. bars} &= 1{\cdot}202 \text{ in.}^2 \\
\text{so overall depth} &= 13{\cdot}45 + {\cdot}44 + 1 \text{ (allowing } 1'' \text{ cover)} \\
&= 14{\cdot}89''
\end{aligned}
$$

Make beam $10'' \times 15''$ overall with two $\frac{7}{8}''$ bars.

(*See* Fig. 7).

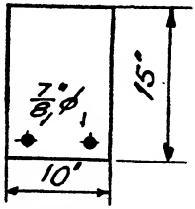


FIG. 7

EXERCISES

1. Given that in a beam the values of c, t and m are respectively 700, 17,500 and 16, find the values of n, a and Q in terms of the effective depth, also the percentage of reinforcement required.

2. A beam is 12 inches wide and has an overall depth of 36 inches. The reinforcement consists of four bars placed $1\frac{1}{2}$ inches from the underside. Using the values of $m=14$, $t=18{,}000$ and $c=750$, find the M.R. of the beam and the diameter of the bars.

3. A concrete slab is continuous over several spans of 10 feet each and carries a superimposed load of 250 lb. per square foot. Find the depth of slab required and the amount of reinforcement. $m=14$, $t=18{,}000$, $c=750$.

4. A slab 12 feet by 6 feet is supported on four edges and carries a load of 3 cwt. per square foot. Find the depth required and the amount of reinforcement in each direction (use Grashof and Rankine formula). $m=15$, $t=16{,}000$, $c=600$.

ANSWERS

1. $n=0{\cdot}39d$; $a=0{\cdot}87d$; $Q=118$; $p=0{\cdot}78$.
2. 1,750,000 in. lbs.; 1 inch dia.
3. 7 inches; $\frac{1}{2}$-inch bars at 6-inch centres.
4. $5\frac{1}{2}$ inches; $\frac{1}{2}$ inch at 8-inch centres over short span.
   $\frac{1}{4}$ inch at 7-inch „ „ long „

Chapter V

TEE BEAMS; ELL BEAMS; DOUBLE REINFORCEMENT

Beams with Compression Reinforcement

WHEN floors and roofs are formed of concrete slabs, it is generally found to be economical to carry the slab on beams of T-section (L-section on the outside wall faces). Since the ribs are cast monolithic with the slab the latter will act as a compression flange. In other words, the slab forms the table or flange of the T- or L- beam.

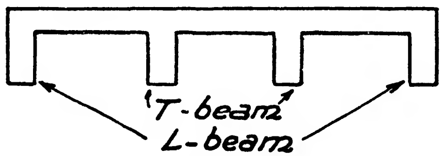


FIG. 8

The breadth of the slab which acts in conjunction with the beam is rather indefinite and different authorities may give varying figures.

The following rule may be taken as representative of modern practice: the least of the three following (for T-beams).

(1) One-third of the effective span of the T-beam.

(2) The centres of the T-beams.

(3) The breadth of the rib plus 12 times the slab thickness.

For L-beams the rule is that the least of the following is taken:

(1) One-sixth of the effective span of the L-beam.

(2) The breadth of the rib plus half the distance between the ribs (not centres of ribs).

(3) The breadth of the rib plus 4 times the slab thickness.

DESIGN OF TEE BEAMS

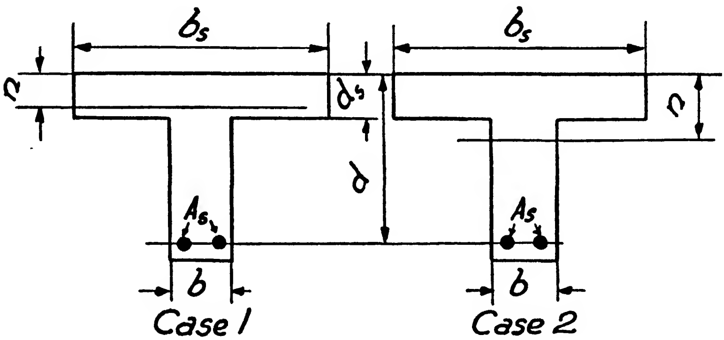


FIG. 9

American regulations for T-beams are that the effective flange width is the least of

(1) One-fourth of effective span length.

(2) Width of rib plus 16 times the slab thickness.

(3) Half the clear distance between ribs plus rib thickness.

American regulations for L-beams are that effective flange width is the least of

(1) One-twelfth of effective span length.

(2) Width of rib plus 6 times the slab thickness.

(3) Half the clear distance between ribs plus the rib thickness.

In the case of T-beams the theory is similar to that of rectangular beams, but two cases must be considered.

Case 1.—Neutral axis comes inside the slab thickness, *i.e.* n is less than d_s (the thickness of the slab). In this

case we substitute b_s the effective breadth of the slab for b and proceed as for ordinary beams. The limit of this case is reached when $n=d_s$.

Case 2.—In this case n is greater than d_s and part of the rib is in compression. It is usual in design to neglect the compression in the rib.

For equilibrium the compression in the concrete must be balanced by the tension in the steel

$$\therefore\ b_s \times d_s \times c_1 = t \times A_t$$

$$= m \times A_t \times \text{(concrete stress at } d \text{ from top)}$$

$$\text{But } \frac{\text{concrete stress at } d}{c} = \frac{d-n}{n}$$

$$\left(c_1 = \text{average stress in slab} = \frac{c}{n} \times \left(n - \frac{d_s}{2}\right)\right)$$

$$\therefore\ b_s \times d_s \times c_1 = m \times A_t \times c \times \frac{d-n}{n}$$

$$\therefore\ \frac{b_s \times d_s}{n} = mA_t\left(\frac{d-n}{n}\right)\left(\frac{2}{2n-d_s}\right)$$

$$\therefore\ (2n-d_s)b_s d_s = 2mA_t(d-n)$$

$$\therefore\ n = \frac{2mA_t d + b_s d_s^2}{2(b_s d_s + mA_t)}$$

The value of a is approximately given by $a = d - \frac{d_s}{2}$

Example 1.—Take the case of a T-beam in which the slab is 4 inches thick and has an effective breadth of 60 inches, the depth to the reinforcement being 16 inches and the area of reinforcement 4·5 square inches. If $c = 750$ lb./in.^{2}, $t = 18{,}000$ lb./in.^{2} and $m = 18$, find the moment of resistance of the section.

Using the formula given above

$$n = \frac{2 \times 18 \times 4{\cdot}5 \times 16 + 60 \times 4^2}{2(60 \times 4 + 18 \times 4{\cdot}5)}$$

$$= \frac{16 \times 222}{2 \times 321} = 5{\cdot}55 \text{ in.}$$

In this case the neutral axis falls within the rib.

$\therefore$ Average stress in the concrete c_1

$$= \frac{750}{5{\cdot}55} \times \left(5{\cdot}55 - \frac{4}{2}\right)$$

$$= 480 \text{ lb./in.}^2$$

Total compression in concrete $= 480 \times 60 \times 4$
$= 115{,}200$ lb.

Using approx. value $a = d - \frac{d}{2}$

$= 16 - \frac{4}{2} = 14$ in.

Moment of Resistance of Concrete $= 115{,}220 \times 14$
$= 1{,}610{,}000$ in. lbs.

Check Moment of Resistance of Steel
$= 4{\cdot}5 \times 18{,}000 \times 14$
$= 1{,}134{,}000$ in. lbs.

This, being the lesser value, will be adopted.

Example 2.—Take the values as in Example 1 but use $A_t = 2{\cdot}0$ sq. in. instead of $4{\cdot}5$ sq. in. Find M.R. of section.

$$n = \frac{2 \times 18 \times 2 \times 16 + 60 \times 4^2}{2(60 \times 4 + 18 \times 2)}$$

$$= \frac{16(72 + 60)}{2 \times 276} = 3{\cdot}82''$$

In this case the neutral axis falls within the slab so we proceed as for a rectangular beam.

$$a = 16 - \frac{3 \cdot 82}{3} = 16 - 1 \cdot 27 = 14 \cdot 73 \text{ in.}$$

$$\text{M.R.}_c = \frac{60 \times 750 \times 3 \cdot 82}{2} \times 14 \cdot 73$$

$$= 1{,}260{,}000 \text{ in. lbs.}$$

$$\text{M.R.}_t = 18{,}000 \times 2 \times 14 \cdot 73 \text{ in.}$$

$$= 530{,}000 \text{ in. lbs.}$$

This will be adopted as being the lesser value.

It will be noted that in the above calculations the breadth of the rib has not been mentioned. Where the neutral axis falls inside the rib, it is usual to neglect the portion of the rib which is in compression. The breadth of the rib is determined by practical considerations usually

(1) the amount of reinforcement used,

(2) the shearing stresses, which are taken up in the rib and not in the slab. We shall refer to the shearing stresses in Chapter VI.

The design of L-beams follows the same lines as the design of T-beams, except that the appropriate values for the breadth of the slab should be used.

It should be noted that in all cases of singly reinforced beams, whether they are rectangular or of flanged section, it is usual to provide "hanger" bars near the top of the concrete. These bars are neglected in the calculations for the strength of the beam, but they are useful for fixing the stirrups (or shear reinforcement) which are usually required. In general, two of these bars will be sufficient for construction purposes. In cases where there is compression reinforcement they are, of course, omitted.

DOUBLY REINFORCED BEAMS.—In the case where the depth available for construction is restricted by headroom or any other cause, the stress developed in the concrete may become more than that permissible in ordinary-grade concrete. When that happens, either a higher-grade concrete can be used, giving higher working stresses, or compression reinforcement can be used. Another reason why it may be good design to use compression reinforcement is that, when the construction depth is limited, the tension reinforcement may be so large that the width of the beam may have to be increased to accommodate this beyond an economical or practical dimension. The saving in the tensile reinforcement is not large as the lever arm is only increased by a small amount. On the whole, it can be stated that double reinforcement of a beam is not economical design, although a certain amount of concrete may be saved by this method. At the same time, practical considerations may lead to the adoption of this system even where the construction depth is not limited. For example, considering continuous beams, over the supports, the bending moments are negative, *i.e.* tension occurs at the top of the section, so that the double system of reinforcement may lend itself well to this condition and simplify the detailing of the reinforcement.

Method I.—Using the same method as for singly reinforced beams, we proceed as follows where

A_c = area of compressive reinforcement
d_c = depth from top of beam to compressive reinforcement
c_s = compressive stress in steel.

Now the total compression is the sum of the compression in the concrete plus the compression in the steel.

Proceeding as before on the theory that the strain is proportional to the distance from the neutral axis and

that the stress is proportional to the strain we get

$$\frac{\text{compression strain on top surface of concrete}}{\text{compression strain on steel}} = \frac{n}{n - d_c}$$

It should be noticed that usually the compressive stress on the concrete at the top of the beam will be the limiting factor in the design of the beam.

As we have seen before, compressive stress on steel = compressive strain on steel × Young's Modulus for Steel (E_s).

$$\therefore \frac{\text{Compressive stress on concrete}}{\text{Compressive stress on steel}} = \frac{E_c \times \text{compressive strain on concrete}}{E_s \times \text{compressive strain on steel}}$$

$$= \frac{1}{m} \times \frac{n}{n - d_c}$$

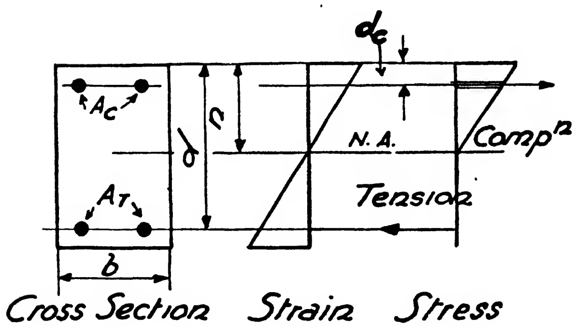


FIG. 10

If we denote compressive stress on concrete by c as usual we find

$$\text{compressive stress on steel} = mc\left(\frac{n - d_c}{n}\right)$$

$$= mc\left(1 - \frac{d_c}{n}\right) \quad . \quad . \quad (1)$$

As in the case of singly reinforced beams we get

$$\frac{n}{d - n} = \frac{c}{\text{tension in concrete at } d}$$

$$\text{Hence } t = \frac{c(d - n)}{n} \times m \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It will be noticed from equation (1) that the compressive stress in the steel is governed by the compressive stress c in the concrete. Assuming c at 750 lb. per square inch and m at 18, and neglecting the small amount d_c, the compressive stress in the steel would be $= 18 \times 750 = 13{,}500$. The allowable stress in the steel being higher than that, it will be seen that the use of compressive reinforcement is not really economical. As we have stated, the total compressive force is the sum of the compression in the steel plus that in the concrete, *i.e.*

$$A_c \times mc\left(1 - \frac{d_c}{n}\right) + \frac{c \times b \times n}{2}$$

The tension in the tensile reinforcement is $A_t \times t$ as before. For equilibrium the two forces must balance

$$\therefore A_t \times t = A_c \times mc\left(1 - \frac{d_c}{n}\right) + \frac{bcn}{2}$$

Substituting for t from (2) we get

$$A_t \times c\frac{(d - n)}{n} \times m = A_c \times mc\left(1 - \frac{d_c}{n}\right) + \frac{bcn}{2}$$

or

$$A_t \times \frac{(d - n)m}{n} = A_c \times m\left(1 - \frac{d_c}{n}\right) + \frac{bn}{2}$$

$$\therefore 2A_t \times m(d - n) = 2A_c \times m(n - d_c) + bn^2 \qquad (3)$$

From this equation (3) n can be found knowing the values of m, A_s, A_c, d and d_c, although the solution is rather involved. Having found the value of n, the next step is to find the value of the lever arm a. This can be done most conveniently by taking moments about the compression edge. The area in compression is $A_c + bn$ and

the compression forces $= A_c \times mc\left(1 - \frac{d_c}{n}\right) + \frac{bcn}{2}$ and

the moment of these forces about the compressed edge

$$= A_c \times mc\left(1 - \frac{d_c}{n}\right) \times d_c + \frac{bcn}{2} \times \frac{n}{3}$$

$$= A_c \times mc\left(1 - \frac{d_c}{n}\right)d_c + \frac{bcn^2}{6}$$

Therefore the depth of the centre of compression is

$$x = \frac{c\left\{A_c \times md_c\left(1 - \frac{d_c}{n}\right) + \frac{bn^2}{6}\right\}}{c\left\{A_c \times m\left(1 - \frac{d_c}{n}\right) + \frac{bn}{2}\right\}}$$

$$= \frac{A_c \times md_c\left(1 - \frac{d_c}{n}\right) + \frac{bn^2}{6}}{A_c \times m\left(1 - \frac{d_c}{n}\right) + \frac{bn}{2}}$$

Then when x is found by inserting the values of A_c, m, d_c, n, the lever arm $a = d - x$.

As the tensile force is $A_t \times t$, the moment of resistance of the beam $= A_t \times t \times (d - x)$

or $$= (d - x) \times c\left\{mA_c\left(1 - \frac{d_c}{n}\right) + \frac{bn}{2}\right\}$$

It will be seen from the foregoing that the calculation of a beam in this way is rather involved and it is common practice to assume that the compression in the steel acts at the same level as the compression in the concrete, *i.e.* at $\frac{n}{3}$ from the top of the beam.

Then compressive stress in the steel $= \frac{2}{3}mc$

$$\text{Total compressive force} = A_c \times \frac{2}{3}mc + \frac{bcn}{2}$$

$$= A_t \times t$$

$$= A_t \times mc\left(\frac{d-n}{n}\right)$$

$$\therefore \frac{2}{3}A_c \times m + \frac{bn}{2} = A_t \times m\left(\frac{d-n}{n}\right) \quad . \quad . \quad (4)$$

or $$4A_cmn + 3bn^2 = 6A_tm(d-n)$$

From which n can be obtained.

Then moment of resistance of beam

$$= c\left(d-\frac{n}{3}\right)\left\{mA_c\left(1-\frac{d_c}{n}\right)+\frac{bn}{2}\right\}$$

$$= c\left(d-\frac{n}{3}\right)\left\{mA_c \times \frac{2}{3}+\frac{bn}{2}\right\} \quad . \quad . \quad (5)$$

or $$= A_t \times t\left(d-\frac{n}{3}\right) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

It will be noticed that the value of the compressive area is too high as the factor mA_c should actually be $(m - 1)A_c$, but the error involved is small and is generally not taken into account in the calculations. It will also be seen that the compression steel does not develop its full stress and

that to develop a force equal to that of the tensile reinforcement considerably more steel would be required.

Method II.—In this case we find the moments of resistance of the steel and of the concrete and add them to find the moment of resistance of the beam.

(1) Moment of Resistance of steel $= A_c \times (d - d_c) \times$ stress in compressive reinforcement.

As we have proved in Method I stress in compressive reinforcement $= \left(1 - \frac{d_c}{n}\right)(m - 1)c$ (applying the correction $(m - 1)$ instead of (m).

$\therefore$ M.R. of compression steel

$$= A_c \times (d - d_c) \times \left(1 - \frac{d_c}{n}\right)(m - 1)c$$

(2) Moment of Resistance of concrete $= \frac{bcn}{2} \times \left(d - \frac{n}{3}\right)$

$$= Qbd^2$$

Therefore the Total Moment of Resistance of the beam

$$= A_c \times (d - d_c)\left(1 - \frac{d_c}{n}\right)(m - 1)c + Qbd^2.$$

It will be remembered * that the Moment of Resistance of a beam with tensile reinforcement only $= Qbd^2$, therefore the additional M.R. is the value of $A_c(d - d_c)\left(1 - \frac{d_c}{n}\right)$ $(m - 1)c$.

In the case where the values of b and d are limited by practical considerations and a certain bending moment has to be resisted we can proceed as follows: find Q from the values of b, c and m, then find the difference between the bending moment applied to the beam and the value of

* See Method for Singly Reinforced Beams.

Qbd^2. This difference has to be taken up by the steel reinforcement.

$$M = Qbd^2 + A_c(d - d_c)\left(1 - \frac{d_c}{n}\right)(m - 1)c$$

Having found n for the beam A_c can be found by substitution of the values for d and d_c. To provide a moment of resistance in tension equal to that in compression, we proportion the value of A_t so that the maximum allowable stress is not exceeded.

As the lever arm is the same, the tensile force must equal the compressive force.

$$A_t \times t = A_c\left(1 - \frac{d_c}{n}\right)(m - 1)c + \frac{bcn}{2}$$

$$\therefore A_t = \frac{A_c\left(1 - \frac{d_c}{n}\right)(m - 1)c + \frac{bcn}{2}}{t}.$$

$$= \frac{A_c\left(1 - \frac{d_c}{n}\right)(m - 1) + \frac{bn}{2}}{\frac{t}{c}}$$

In the case where the value of A_c exceeds the value of A_t, we can apply what is known as the "steel beam theory." Then the moment of resistance will be $tA_t(d - d_c)$ as we make $A_t = A_c$.

It should be noted that in this case where A_c is equal to A_t that the allowable stress in the concrete may be exceeded as

$$A_c = \frac{t \times A_t}{\text{stress in compressive steel}} = A_t$$

$$\text{but } \frac{\text{stress in compressive steel}}{\text{max. stress in concrete}} = \frac{n - d_c}{n} \times (m - 1)$$

$\therefore$ max. stress in concrete

$$= \frac{\text{stress in compressive steel} \times n}{(m-1)(n-d_c)}$$

$$= \frac{\dfrac{A_t \times t}{A_c} \times n}{(m-1)(n-d_c)} = \frac{t \times n}{(m-1)(n-d_c)}$$

Assuming $d_c = \dfrac{n}{3}$

$$\text{max. stress in concrete} = \frac{3(t)}{2(m-1)}$$

For example, if t is 18,000 and $m = 18$

$$\text{stress in concrete} = \frac{27{,}000}{17}$$

$$= 1{,}580 \text{ lb./in.}^2$$

This is above the allowable working stress for ordinary concrete.

American practice is somewhat different from British with regard to compression reinforcement. American regulations allow a stress in compression reinforcement of twice the value given by $\left\{(m-1)\, c \left(\dfrac{n-d_c}{n}\right)\right\}$ provided that the maximum stress given by this rule does not exceed 16,000 lbs./in.^{2} For a graphical solution of the somewhat complicated design of doubly reinforced concrete beams (rectangular and T-beams) *see* the articles by Mr. E. G. S. Powell in the *Structural Engineer* of February, 1942, and June, 1943, on "Design of Doubly Reinforced Beams."

Example 1.—Using Method I and assuming that $d_c = \frac{n}{3}$, find the M.R. of a rectangular beam with $b = 10$ inches, $d = 20$ inches, $A_t = 3{\cdot}14$ square inches and $A_c = A_t$.

Take $t = 18{,}000$, $c = 750$ and $m = 18$

First we must find the value of n from (4)

$$\frac{2}{3} \times 3{\cdot}14 \times 18 + \frac{10n}{2} = 3{\cdot}14 \times 18\left(\frac{20 - n}{2}\right)$$

$$\therefore \quad 12 + 1{\cdot}59n = 18\left(\frac{20 - n}{n}\right)$$

$$12n + 1{\cdot}59n^2 = 360 - 18n$$

$$1{\cdot}59n^2 + 30n - 360 = 0$$

$$\therefore\ n = 8{\cdot}2 \text{ in.}$$

Moment of Resistance of beam in compression

$$= 750\,(20 - 2{\cdot}7)\left\{\left(18 \times 3{\cdot}14 \times \frac{2}{3}\right) + \frac{10 \times 8{\cdot}2}{2}\right\}$$

from (5)

$$= 750 \times 17{\cdot}3\,(37{\cdot}68 + 41)$$

$$= 13{,}000 \times 78{\cdot}68$$

$$= 1{,}000{,}000 \text{ in. lbs.}$$

Moment of Resistance of beam in tension

$$= 3{\cdot}14 \times 18{,}000 \times 17{\cdot}3 \text{ from (6)}$$

$$= 975{,}000 \text{ in. lbs.}$$

Example 2.—Using Method II with b, d, A_t, t, c and m as in Example 1, find the area A_c so that the M.R. of the beam may be 1,500,000 in. lbs.

M.R. of singly reinforced beam $= Q \times bd^2$
(in compression) $= 138 \times 10 \times 20^2$
$= 552{,}000$ in. lbs.

Extra M.R. to be provided $= 1{,}500{,}000 - 552{,}000$

$= 948{,}000$ in. lbs.

$n = 8\cdot 6$ in. $= A_c(20-2\cdot 9)(\frac{2}{3})17 \times 750$

$= A_c \times 17\cdot 1 \times 8{,}500$

$= 145{,}000\ A_c$

$\therefore A_c = 6\cdot 4$ sq. in.

As this is more than double the value of A_t the design is not economical.

Example 3.—Using the "steel beam theory" and with $A_t = 6$ square inches, $A_c = 6$ square inches, $t = 18{,}000$ $d = 30$ inches and $d_c = 2$ inches, find the moment of resistance of the beam. Find the maximum compressive stress in the concrete.

M.R. $= 18{,}000 \times 6 \times 28$

$= 3{,}040{,}000$ in. lbs.

Now $n = 0\cdot 43d = 12\cdot 9$ in.

$$\therefore \text{max. stress } (c) \text{ in concrete} = \frac{18{,}000 \times 12\cdot 9}{17 \times 10\cdot 9}$$

$$= 1{,}250 \text{ lb./in.}^2$$

This is above the safe stress for ordinary concrete.

DOUBLY REINFORCED T-BEAMS.—When a T-beam is continuous over two or more spans, it must be remembered that the bending moments over the supports are negative, *i.e.* they cause tension at the top of the beam. In this case, the slab does not come into play as it is put into tension and the rib only acts in compression. For this reason, it may be an advantage to have compressive reinforcement or to bend the reinforcing bars up over the supports.

The treatment of doubly reinforced T-beams is very similar to that for rectangular beams. Proceeding in the

same manner as before and taking moments to find the value of n we get

$$b_s d_s\left(n - \frac{d_s}{2}\right) + m \times A_c(n - d_c) - mA_t(d - n) = 0$$

$$\therefore n = \frac{b_s d_s^2 + 2m(A_c d_c + A_t d)}{2\{b_s d_s + m(A_c + A_t)\}}$$

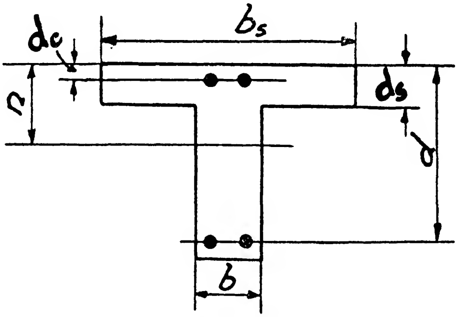


FIG. 11

From which we find n by inserting the values for m, A_c, A_t, d_c, d, b_s and d_s.

$$\therefore I = \text{Moment of Inertia of beam} = \frac{b_s}{3}\{n^3 - (n - d_s)^3\}.$$

$$+ mA_t(d - n)^2 + mA_c(n - d_c)^2 \text{ in concrete units}$$

Then Moment of Resistance $= \dfrac{fI}{y} = \dfrac{cI}{n}$ in compression

or $\qquad = \dfrac{t \times I}{m(d - n)}$ in tension

An approximation which is sometimes used is to assume that the compressive force in the concrete acts at 0·4 times d_s from the top and the compressive reinforcement at the same level.

Another method is as Method II for rectangular beams in which we add the moments of resistance of the steel and the concrete. Some designers work on the "steel beam theory." In using this method it should be borne in mind that the safe working stress for the type of concrete used should not be exceeded. As mentioned before, where a T-beam is continuous over several spans, the bending moment is negative in places and it may be necessary to allow for this by bending the bars up.

Example 1.—The beam shown is subject to a negative B.M. of 5,000,000 in. lbs. Find the amount of compressive

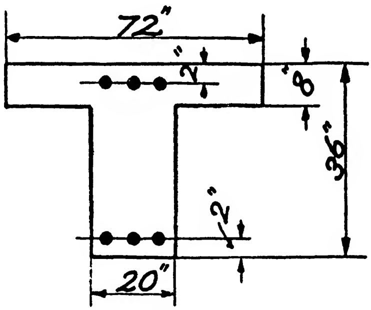


FIG. 12

reinforcement required. (Values of $t = 18{,}000$, $m = 18$, $c = 750$).

The section is equivalent to a beam 20 in. wide and $(36 - 2) = 34$ in. deep.

Now M.R. $= 138 \times 20 \times 34^2$.
$= 3{,}150{,}000$ in. lbs.

Bending Moment to be taken up by compressive reinforcement

$= 5{,}000{,}000 - 3{,}150{,}000$
$= 1{,}850{,}000$ in. lbs.

Lever arm for compressive reinforcement $= 34 - 2$
$= 32$ in.

Allowable stress in compressive steel

$$= \frac{18 \times 750 \times (0{\cdot}43 \times 34 - 2)}{0{\cdot}43 \times 34}$$

$= 11{,}500$ lb. per sq. in.

$\therefore$ Area of compressive reinforcement required

$$= \frac{1{,}850{,}000}{32 \times 11{,}500}$$

$= 4{\cdot}5$ sq. in.

8 — $\frac{7}{8}$-in. dia. bars give an area of $4{\cdot}81$ sq. in., so this reinforcement can be used.

EXERCISES

1. A T-beam has the following dimensions: $b_s = 60$ inches, $d = 30$ inches, $d_s = 6$ inches and $A_t = 10$ square inches. Using $m = 18$, $c = 750$, $t = 18{,}000$ find the bending moment which it will carry.

2. A beam 10 inches wide and 30 inches effective depth has $A_t = 6$ square inches. Using the same values as (1) find what A_c is required so that the M.R. is 3,000,000 in. lbs.

3. Using "steel beam theory" and $A_t = 10$ square inches $= A_c$, $t = 18{,}000$, $m = 18$, $d = 48$ inches and $d_c = 3$ inches, find Moment of Resistance and stress in concrete.

4. A T-beam is subjected to a negative B.M. of 3,000,000 in. lbs. If the rib is 12 inches wide and the overall depth is 30 inches, the cover being 2 inches to the centre of the reinforcement top and bottom, find the value of A_t for $t = 18{,}000$, $c = 750$ and $m = 18$.

ANSWERS

1. n = 12 in., M.R. = 5,450,000 in. lbs. comp.
= 4,860,000 ,, tension.

2. 7·8 square inches.

3. M.R. = 8,100,000 in. lbs., c = 1,160 lb. per square inch.

4. A_c = 5·70 square inches.

Chapter VI

SHEAR STRESSES IN BEAMS

Shear Reinforcement; Bond Stresses; Anchorage; Hooked Ends

SHEAR STRESSES IN BEAMS.—*Theoretical treatment of shear in beams.* Shear usually varies along the length of any loaded

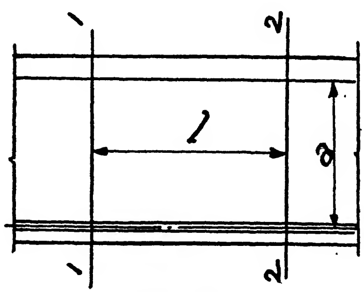


FIG. 13

beam and the shear at any point can be related to the Bending Moment. Suppose in the case of a beam (Fig. 13) the Bending Moment at section 1 — 1 is M_1 and at section

2 — 2 is M_2 (Beam uniform in section). Then if a be the lever arm at any section, the compressive force in concrete

$$= \text{tensile force in steel}$$

$$= \frac{\text{Moment}}{a}$$

∴ Tensile force at 1 — 1 = compressive force at 1 — 1

$$= M_1/a.$$

Similarly

tensile force at 2 — 2 = compressive force at 2 — 2

$$= M_2/a.$$

∴ Tensile force at (2 — 2) — tensile force at (1 — 1)

$$= \frac{M_2 - M_1}{a}$$

∴ The tensile force has increased between 1 — 1 and 2 — 2 (M_2 being greater than M_1), by an amount equal to the change in Bending Moment divided by the lever arm. Now this force $\frac{M_2 - M_1}{a}$ is tending to pull the steel out of the concrete and this tendency is resisted by the bond between the steel and the concrete. Let l be the distance between sections 1 — 1 and 2 — 2 and O be the perimeter or sum of the perimeters of the reinforcing bars, then the force $O \times l \times$ bond stress must equal $\frac{M_2 - M_1}{a}$. If the bond stress s_b exceeds the allowable stress for the concrete used the bars will tend to pull out of the concrete.

$$s_b = \frac{M_2 - M_1}{a \times l \times O} \quad . \quad . \quad . \quad . \quad (1)$$

Allowable values of s_b can be taken as follow:

Mix.	s_b	
1 : 1 : 2	123	lb. per sq. in.
1 : 1·2 : 2·4	115	
1 : 1·5 : 3	110	
1 : 2 : 4	100	

Now since the Bending Moment diagram is the "sum curve" of the Shear Force diagram, the shear is equal to the rate of change of Bending Moment which is $M_2 - M_1/l$.

If S is the transverse shear force on the beam at the section under consideration then

$$S = \frac{M_2 - M_1}{l} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

But from (1) $\dfrac{M_2 - M_1}{l} = s_b \times a \times O$

$\therefore S = s_b \times a \times O.$

If the case of rectangular beams with single reinforcement $a = d - \dfrac{n}{3}$ (this is also the case where compressive reinforcement is assumed to act at $n/3$ from the top, or in T-beams where the neutral axis falls within the slab).

Then $$S = s_b \times \left(d - \frac{n}{3}\right) \times O.$$

It should be noted that this shear is the horizontal shear only which determines the amount of the bond stress.

VERTICAL SHEAR.—Let s be shear stress in lb. per square inch. We have shown that $S = s_b \times \left(d - \dfrac{n}{3}\right) \times O.$

This shear force S acts on the area $a \times b$ (where b is the width of the beam as before)

$$\therefore s = \frac{S}{ab} = \frac{s_b \times \left(d - \frac{n}{3}\right) \times O}{a \times b} \quad . \quad . \quad (3)$$

$$= \frac{M_2 - M_1}{l \times a \times b} \quad . \quad . \quad . \quad . \quad (4)$$

Now considering Fig. 14, taking a cube with side equal to one unit of length and having a unit shear horizontally of s then there is a couple equal to $s \times 1$ tending to rotate the unit of area in a clockwise direction. Now to balance this tendency there must be a couple tending to rotate the unit of area in an anti-clockwise direction equal to $s \times 1$. Therefore we find that the vertical shear must be equal to the horizontal shear. This is an important fact which applies to all structures. Having found the shearing stress from the Bending Moment or Shear Force diagram

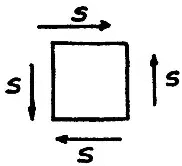


FIG. 14

we must find whether this exceeds the allowable shearing stress for the grade of concrete used. Where the safe shearing stress is exceeded, shear reinforcement must be provided. This can be done in several ways,

(1) by vertical bars known as stirrups,

(2) by bending up the tensile reinforcement usually at 45°,

(3) by a combination of (1) and (2).

ALLOWABLE STRESSES IN STIRRUPS.—Modern practice requires the spacing of stirrups to be not greater than the lever arm or 12 times the diameter of the main bars. The cross-sectional area can be calculated from the formula

$$S = \frac{t_w \times A_w \times a}{p}$$

Where t_w = tensile stress allowable in shear reinforcement

A_w = cross-sectional area of stirrup

p = pitch or spacing of stirrups

t_w can be taken as 18,000 lb. per square inch. It should be noted that the area A_w is usually twice the area of circle of same diameter as the stirrups. The stirrups or links should be secured to the main reinforcement. (Where there is no compressive reinforcement, they will be fixed to the hanger bars).

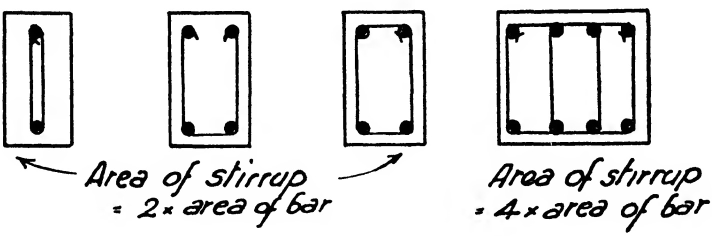


FIG. 15

SPACING OF STIRRUPS.—As the shear is usually greatest nearest the ends of a span the spacing of stirrups will be closest at the ends and will increase towards mid-span. In cases where a beam is doubly reinforced it is good practice to provide stirrups throughout the whole length to avoid buckling of the compressive reinforcement. The minimum size for stirrups should be not less than ¼ inch. Generally bars more than ½ inch diameter are difficult and costly to use on account of bending. It should be noted that when a doubly reinforced beam is calculated on the "steel beam" theory, the spacing of the stirrups should be not greater than 6 inches or 8 times the diameter of the main bars and the diameter of the stirrups should be calculated to suit this.

FORMS OF STIRRUPS.—These may be various to suit the main reinforcement used.

DIAGONAL REINFORCEMENT.—To resist the heavy shear towards the ends of a span, it is common practice to bend up the tensile reinforcement. Usually the Bending Moment decreases from the mid-span towards the ends of the span. If we investigate the B.M. we find that towards the ends of some of the spans the tensile reinforcement can be omitted (B.M. $= A_t \times a \times t$). Then the point at which a bar can be omitted is called its "theoretical end" and beyond this point it can be bent up to form shear reinforcement. It is good practice to bend up the bars in pairs where there are a number of bars. In the case where there are several tiers of bars forming the tensile reinforcement, the top tier can be bent up, then the next tier and so on. The bottom tier should be carried to the ends of the span as in the case of continuous beams it can form the compressive reinforcement for the negative bending moment and also acts as stirrup anchorage. The diagonal reinforcement acts in tension and the vertical component resists this tension.

If t = allowable tensile stress in bent-up bar

θ = angle of inclination

A = area of bar

Then force resisting shear $= At \times \sin\theta$

If θ is 45° as is common practice this becomes $\dfrac{At}{\sqrt{2}}$.

Where there are several bars the area A will be sum of the areas of bars. It is interesting to note that the diagonal bars are the equivalent of the inclined bars of a lattice girder in steel-work. The allowable stress t can be taken as 18,000 lb. per square inch as in the case of stirrups.

PRACTICAL CONSIDERATIONS IN THE DESIGN OF SHEAR REINFORCEMENT.—It is important to pay as much attention to the design of the shear reinforcement as to the design of the main reinforcement to resist the bending moment. It has been shown that horizontal shear stresses set up vertical shear stresses of the same magnitude. Now as there are two forces at right angles (Fig. 14), taking the sides of the unit square in pairs, we find that tensile forces exist along one diagonal and compressive forces along the other diagonal. The compressive forces can be ignored as if shear stress be s then the diagonal stress will be $s \times \sqrt{2}$. As s is generally taken as $c/10$ then the maximum diagonal compressive stress will be only $0 \cdot 14\ c$. The tensile stress will also be $0 \cdot 14\ c$. The permissible diagonal tensile stress will be in the nature of $c/10$ so it is obvious that diagonal reinforcement is important.

The first consideration is to find the shear stress in lb. per square inch on the section and to find whether this exceeds the permissible stress. Where the actual stress exceeds the permissible it is the practice of some designers to neglect the strength of the concrete in shear entirely and to take all the shear on the reinforcement. On the other hand, some designers tend to stress the concrete in shear to the limit. An intermediate method is to allow the full permissible value of shear stress on the concrete, when this is not exceeded by the actual stress, and to reduce this value when the actual stress exceeds the tabulated safe stress. Thus when s (actual) $=$ s (allowable) we use the s allowable in calculating the shear strength of the member, but when s (actual) exceeds s (allowable) we reduce s (allowable) in proportion.

For example, where s (actual) $= 2 \times s$ (allowable) the value of s (allowable) is reduced by 50 per cent and so on. In cases where s (actual) $= 4 \times s$ (allowable) the shear

strength of the concrete should be neglected altogether.

It is obvious from above that in all cases where the actual shear stress is greater than the permissible, the excess shear should be taken up by shear reinforcement in one form or another.

The method adopted can best be seen by considering one or two typical cases.

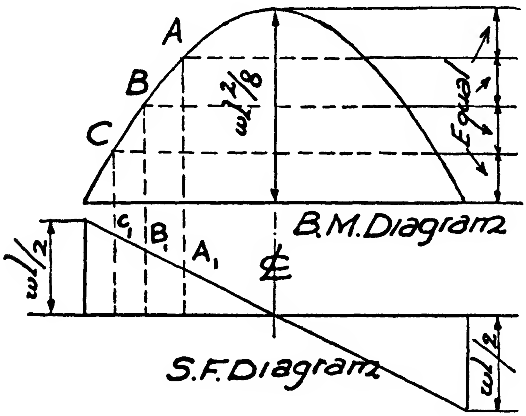


FIG. 16

Case 1.—Beam uniformly loaded. In this case the maximum B.M. for a simply supported beam will be $\frac{wl^2}{8}$, the diagram being parabolic, and the shear will be $\frac{wl}{2}$ at each end of the span and zero in the middle.

The B.M. diminishes towards the ends of the span (see Fig. 16). Now the M.R. of the beam is $A_t \times t \times a$ and since a and t can be taken as constant therefore the value of A_t should diminish from mid-span to the ends of the span. From the diagram we can find the theoretical ends

of the bars and mark these on the diagram. (Assume 4 tiers of tensile reinforcement). Project the points A, B, C on to the shear force diagram and find the value of the shear force at these points. The shear between A_1 and B_1 will be resisted by the bars in the top tier. The shear between A_1 and C_1 will be resisted by the bars in the top and 2nd tiers and the shear between A_1 and the support will be resisted by the bars in the top and 2nd and 3rd tiers. Now consider the shear at A_1. If the shear stress is more than the allowable we must provide stirrups to take up the excess shear. Having fixed on a suitable diameter and form of stirrup, we find the spacing from the formula given and space them accordingly. It should be noted that stirrups should be provided between A_1 and the support for constructional purposes.

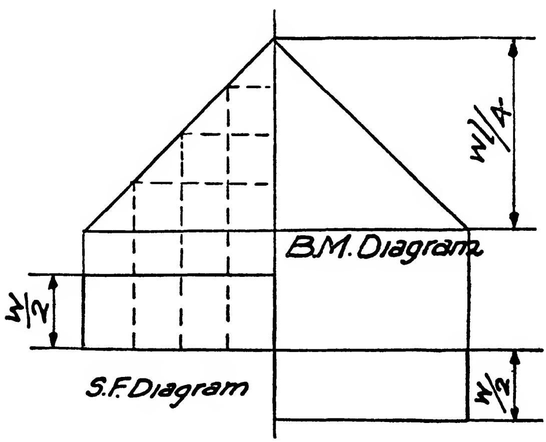


FIG. 17

Case 2.—Beam with a point load W at mid-span. In this case the maximum B.M. will be Wl/4 and shear will be W/2.

In this case the shear is constant from the support to

mid-span, changing sign there. The procedure is, however, the same as before. Some designers use spacing coefficients for stirrups in such standard cases, but in the case of important beams it is best to investigate the shearing stresses from the B.M. and S.F. diagrams. In secondary beams it is sufficient to find where shear reinforcement can be omitted and the maximum shear reinforcement required. The intermediate reinforcement can be spaced between these points at the discretion of the designer.

Example 1.—A beam 20 feet span carries a load of 500 lb. per foot run. Design the tensile and shear reinforcement.

$$\text{Max. B.M.} = \frac{wl^2}{8} = \frac{500 \times 20^2 \times 12}{8}$$

$$= 500 \times 600 = 300{,}000 \text{ in. lbs.}$$

Take $b = 10 \quad d = 15$.

Now taking $m = 18$, $c = 750$ and $t = 18{,}000$ we get $a = 0{\cdot}86 \times 15 = 12{\cdot}9$

$$\therefore A_t = \frac{300{,}000}{12{\cdot}9 \times 18{,}000} = 1{\cdot}3 \text{ in.}^2$$

Use 8 - $\frac{1}{2}$ in. bars giving an area of $1{\cdot}57$ in.^{2}

$$\text{Now max. shear} = \frac{wl}{2} = \frac{500 \times 20}{2}$$

$$= 5{,}000 \text{ lb.}$$

Allowable shear stress = 75 lb. per sq. in.

$$\text{Actual shear stress} = \frac{5{,}000}{12{\cdot}9 \times 10} = 39 \text{ lb. per sq. in.}$$

Theoretically no shear reinforcement is required.

Example 2.—A beam of same dimensions as Example 1 carries a point load of 30,000 lb. at centre. Find the reinforcement required in shear.

$$\text{Max. B.M.} = \frac{30{,}000 \times 20 \times 12}{4} = 1{,}800{,}000 \text{ in. lbs.}$$

$$A_t = \frac{1{,}800{,}000}{12 \cdot 9 \times 18{,}000} = 7 \cdot 75 \text{ sq. in.}$$

Use 8 - 1 in. dia. bars in two tiers (Area = 7·95 sq. in.)
Max. shear = 15,000 lb.

$$\text{Shear stress} = \frac{15{,}000}{12 \cdot 9 \times 10} = 117 \text{ lb. per sq. in.}$$

Bending the upper tier of the tensile reinforcement up in pairs we get

$$\text{Shear taken by diagonals} = 2 \times \frac{7 \cdot 95}{8} \times 18{,}000 \times \frac{1}{\sqrt{2}}$$

$$= 25{,}000 > 15{,}000$$

Therefore they take all the shear.

Using $\frac{1}{4}$ in. dia. U-shaped stirrups where there are no diagonals we get

$$15{,}000 = \frac{18{,}000 \times 2 \times 0 \cdot 049 \times 12 \cdot 9}{p}$$

$\therefore p = 15 \cdot 4$ in. As this is greater than a the spacing should not exceed 12 in.

ANCHORAGE.—Every main reinforcing bar used in tension and also bent-up bars used to resist shear should extend

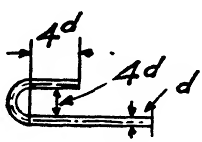


FIG. 17A

sufficiently far beyond their theoretical ends to enable the strength of the bars to be developed by the adhesion of

the concrete on the lengths beyond the theoretical ends. If T be pull in the bar and O the perimeter, the length for developing its strength will then be

$$l = \frac{T}{O \times s_b}$$

where s_b is the bond stress allowable between steel and concrete. End anchorage may usually be effected in two ways,

(1) by straight length

(2) by hooked end.

In the case of tension bars a rule which is often used is that the length beyond the theoretical end = length to develop strength by adhesion + 14 diameters (for straight bars) and = length to develop strength measured on straight (for hooked bars). In the case of bent-up shear bars the length in question can be measured from the neutral axis.

In the case of hooked bars the inside diameter of the hook should be not less than 4 times the diameter of the bar, except where a bar is hooked over another, when it can have an inside diameter equal to the diameter of the bar over which it is hooked.

The length of the straight part of the bar beyond the end of the curve should be not less than 4 times the diameter of the bar. It should be noted that in any case the end anchorage is in addition to the length required to develop the strength of the bar.

American design regulations for treatment of shear is somewhat different from British. Without dealing with this difference at length, it can be stated that American practice is to increase the permissible shear stress in the concrete according to the amount of shear reinforcement and the method of anchorage of the main steel.

EXERCISES

1. The shear at the end of beam 10 inches wide and 20 inches effective depth is 20,000 lb. Find the shear stress and the spacing for ½-inch U stirrups

$$\begin{pmatrix} a = \cdot 86d \\ s = 75 \text{ lb./in.}^2 \end{pmatrix}$$

2. A beam similar to that in (1) has $A_t = 8 - 1\frac{1}{8}$ inch diameter bars. Find what shear the diagonals will resist if bent up at 45° in pairs.

3. A bar 1 inch diameter is stressed to 18,000 lb. per square inch. Find what length will be required to develop its strength when the allowable bond stress is 100 lb. per square inch.

ANSWERS

1. (*a*) 116 lb. per square inch; (*b*) 6 inches.
2. 25,500 lb.
3. $l = 45$ inches.

Chapter VII

COLUMNS AND FOUNDATIONS

Systems of Reinforcement and Stirrups; Working Stresses

CONCRETE columns consist of a concrete core reinforced by a system of main longitudinal bars. When subject to direct load the steel and the concrete are in direct compression and stress in the steel will be m times the stress in the surrounding concrete. The case of columns subjected to bending as well as to direct compression is rather com-

plicated and is beyond the scope of this book. In addition to the main longitudinal reinforcement reinforced concrete columns should have secondary reinforcement to prevent buckling of the main bars and bursting of the concrete outwards. If this is omitted, the column becomes what is known as a "rodded" column and many failures have been due to this fault. The secondary or transverse reinforcement may be

(1) in the form of hoops (similar to the stirrups for beam reinforcement),
(2) in the form of a spiral or helix,
(3) a combination of (1) and (2).

The secondary reinforcement is subject to tension. The area of a column may be calculated as either,

(1) gross area which is the area D × D (Fig. 18) or
(2) core area $d \times d$ shown hatched in Fig. 18.

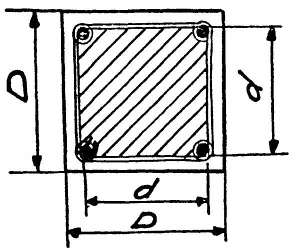


FIG. 18

It should be noted that the minimum cover is 1 inch measured from the outside of the transverse reinforcement. The effective area of a concrete column is the area of the concrete + $(m - 1)$ times the area of the longitudinal (main) reinforcement or $A = D \times D + (m - 1)A_c$. This value A multiplied by the allowable compressive stress fc gives the

safe load for short columns in which crushing takes place before buckling:

$$\text{Safe load} = f c\{D \times D + (m - 1)A_c\}.$$

LONGITUDINAL REINFORCEMENT.—The area of longitudinal steel used for columns should be not less than 0·8 per cent nor more than 8·0 per cent of the gross area of the column.

When helical reinforcement is used there should be not less than 6 bars in the main reinforcement, but when hoops (or binders) are used it is sufficient to have one bar at each corner of the column.

TRANSVERSE REINFORCEMENT.—The volume of transverse reinforcement used should be not less than 0·4 per cent of the gross volume of the column. When binders are used they should be not less than 6 inches or more than 12 inches apart. When the maximum spacing is used the diameter of the binder should be not less than a quarter of that of the bars used as longitudinal reinforcement. For closer pitches the diameter of the binders may be reduced within the limit imposed by the rule of 0·4 per cent for volume. In cases where spiral reinforcement is used the pitch of the spiral should be not more than 3 inches or one-sixth of the core diameter (whichever is the lesser) and not less than 1 inch or three times the diameter of the main reinforcement (whichever is the greater).

Allowable Stress for Concrete in Direct Compression

Mix	Stress in lb. per sq. in.
1 : 1 : 2	780
1 : 1·2 : 2·4	740
1 : 1·5 : 3	680
1 : 2 : 4	600

For columns the allowable compressive stress in main

reinforcement can be taken as 13,500 lb. per square inch and the allowable tensile stress in spiral reinforcement as 13,500. Note that these values are for Mild Steel. Where steel with a yield point of not less than 44,000 lb. per square inch is used the values can be increased to 15,000 lb. per square inch.

LOAD IN COLUMNS (as used in modern practice).

For Short Columns.

(*a*) with binders:

$$\text{Safe load} = f_c \times \text{gross area of concrete} + 13{,}500\ A_c.$$

(*b*) with spiral reinforcement:

$$\text{Safe load} = f_c \times \text{core area of concrete} + 13{,}500\ A_c + 2 \times 13{,}500 \times \text{equivalent area of spiral reinforcement}$$

(equivalent area of spiral reinforcement = volume of spiral divided by length of column).

The reason why a higher load is allowed on columns (*b*) is that tests have shown conclusively that helically bound columns can develop a much higher stress in the concrete core before failing.

These formulæ apply only to columns where there is no bending on the columns.

For Long Columns.

The safe load in this case is obtained by multiplying the load as above by a coefficient depending on the ratio $\dfrac{\text{effective length of columns}}{\text{least lateral dimension}}$. In case (*a*) above the least lateral dimension is the least overall dimension and in case (*b*) the least lateral dimension of the core. The coefficients are as given in the following table (this applies only to columns symmetrical about two axes at right angles).

$\dfrac{\text{Effective length of column}}{\text{least lateral dimension}}$	Coefficient
15	1·0
18	0·9
21	0·8
24	0·7
27	0·6
30	0·5
33	0·4
36	0·3
39	0·2
42	0·1
45	0

The reason for this reduction is to allow for the buckling which occurs in long columns.

L.C.C. By-laws give a coefficient depending on the l/k ratio thus

l/k	Coefficient
50	1·0
60	0·9
70	0·8
80	0·7
90	0·6
100	0·5
110	0·4
120	0·3

k is the least radius of gyration $= \sqrt{\frac{I}{A}}$. In calculating the values of I and A, the area of longitudinal reinforcement must be replaced by $(m - 1)$ times its area of concrete to find the "equivalent" area.

The effective length may be defined as the "equivalent length" of a hinged ended strut or column. The ratio

$\dfrac{\text{effective length}}{\text{actual length}}$ depends on the end conditions of the column, whether they are fixed, hinged or free. It is rather difficult to lay down an exact definition of a fixed or hinged end. The designer must use his own discretion and experience in estimating the amount of fixity.

For Single Storey Columns.

End Conditions	$\dfrac{\text{Effective length}}{\text{Actual length}}$
Both ends fixed	0·75
Both ends hinged	1·00
One end fixed, one end free . .	1·00 - 2·00

For Columns of Two or More Storeys.

Both ends fixed	0·75
Both ends hinged	0·75 - 1·00
One end fixed, one end free . .	1·00 - 2·00

Example 1.—A concrete column 10 inches square (1 : 2 : 4 mix) is 15 feet high and fixed at both ends. The main longitudinal reinforcement consists of 4 - 1 inch diameter bars with binders for lateral reinforcement. Find the safe load for the column.

$$\begin{aligned}
\text{Safe load for short column} &= (600 \times 10 \times 10) + (13{,}500 \times 3{\cdot}14) \\
&= 60{,}000 + 42{,}500 \\
&= 102{,}500 \text{ lb.} \\
\text{Actual length} &= 180 \text{ in.} \\
\text{Effective length} &= 135 \\
\frac{\text{Effective length}}{\text{least lateral dimension}} &= \frac{135}{10} \\
&= 13{\cdot}5 \\
\therefore \text{Coefficient} &= 1{\cdot}0 \\
\therefore \text{Safe load} &= 102{,}500 \text{ lb.}
\end{aligned}$$

Check by L.C.C.

$$A = 10 \times 10 + 14\,(3{\cdot}14) = 100 + 43{\cdot}96$$
$$= 143{\cdot}96 \text{ in.}^2$$

$$I = \frac{BD^3}{12} + 4 \times 14 \times (5 - 1\tfrac{3}{4})^2$$
$$= \frac{10^4}{12} + 56 \times 3{\cdot}25^2$$
$$= 8{,}333 + 595 = 8{,}928 \text{ in.}^4$$

$$k = \sqrt{\frac{8{,}928}{143{\cdot}96}} = 7{\cdot}8$$

$$l/k = \frac{135}{7{\cdot}8} = 17 \qquad \therefore \text{ Coefficient} = 1{\cdot}0$$

Example 2.—A concrete column 12 inches diameter is 20 feet high and is hinged at each end. The mix is 1 : 1·5 : 3 and the longitudinal reinforcement consists of 6 - $1\frac{1}{8}$ inch diameter bars. Find what transverse reinforcement is required and the safe load on the column.

Pitch of helical reinforcement = 2 in. approx.

Minimum dia. of helical reinforcement = $\frac{5}{16}$ in.

Vol. of helical reinforcement per foot length

$$= 6 \times \pi \times 10 \times 0{\cdot}076$$
$$= 1{\cdot}43 \text{ sq. in.}$$

$$\text{Core area} = \frac{\pi}{4}\left\{12 - 2 \times 1\tfrac{1}{2}\right\}^2$$
$$= \frac{\pi}{4}(9)^2 = 64 \text{ sq. in.}$$

$$A_c = 6{\cdot}00 \text{ sq. in.}$$

∴ Safe load for short column

$$= (64 \times 680) + (6 \times 13{,}500) + (2 \times 1{\cdot}43 \times 13{,}500)$$
$$= 43{,}500 + 81{,}000 + 38{,}700$$
$$= 163{,}200 \text{ lb.}$$

Effective length = 240 in.

$$\frac{\text{Effective length}}{\text{least lateral dimension}} = \frac{240}{9} = 26{\cdot}7$$

Coefficient = 0·60

∴ Safe load = 0·60 × 163,200 = 97,920 lb.

Check by L.C.C.

$$A = \pi/4 \times 12^2 + 14 \times 6$$
$$= 113{\cdot}6 + 84 = 197{\cdot}6 \text{ in.}^2$$
$$I = \frac{\pi}{64} D^4 + 4 \times 14 \times 1 \times \left(4{\cdot}5 \times \frac{\sqrt{3}}{2}\right)^2$$
$$= 1{,}020 + (56 \times 20{\cdot}25 \times \tfrac{3}{4})$$
$$= 1{,}020 + 850 = 1{,}870$$
$$k = \sqrt{\frac{1{,}870}{197{\cdot}6}} = 3{\cdot}07$$
$$l/k = \frac{240}{3{\cdot}07} = 78 \qquad \text{Coefficient} = 0{\cdot}72$$

This gives a slightly higher value than before.

Concrete columns may be square, round or octagonal and the number of main reinforcing bars may vary, also the arrangement of hoops. It is generally economical to use the largest size of columns which circumstances permit as concrete is cheaper than steel. Square columns are cheapest to construct as the shuttering is simpler than in the case of round or octagonal columns.

When it is necessary to make a joint in the longitudinal reinforcement this can be done in several ways:

(1) by overlapping the bars for a length of at least 24 times the diameter of the upper bars or a sufficient length to develop the force in the bars by bond (whichever is the lesser),

(2) by welding the bars together

(3) by using screwed couplings.

Where spiral reinforcement is lapped, the amount of overlap should be not less than 40 diameters. It should be noted that the core diameter for helically bound columns is 3 inches less than the outside diameter. When a spirally bound column is adopted the octagonal section is most economical, since it costs less for shuttering than a round section and there is less concrete outside the core than for a square column. It should also be noted that a helically bound column takes a greater load than one with binders and that where construction space is limited it may be an advantage to adopt this system. When a column is required to carry a heavy load it may be economical to use a richer mix of concrete than usual instead of using additional main reinforcement as by so doing the working stress of the concrete may be raised sufficiently to take the load. The maximum size of binders generally used is $\frac{1}{2}$ inch diameter, while $\frac{1}{4}$ inch and $\frac{5}{16}$ inch diameter are the sizes most commonly used. The same remarks apply also to spiral reinforcement.

FOUNDATIONS.—Column foundations may be of two kinds:

(1) where the footing is composed of reinforced concrete and rests directly upon the subsoil,

(2) where the footing rests upon a block of unreinforced concrete which rests upon the subsoil.

Whatever type of foundation is used the pressure on

the subsoil should not exceed the following values:

	Tons per sq. ft.
Alluvial soil, made ground, very wet sand	½
Soft clay, wet or loose sand	1
Ordinary fairly dry clay, dry fine sand, sandy clay	2
Firm dry clay	3
Compact sand or gravel, London blue clay	4
Hard solid chalk	6
Shale and soft rock	10
Hard rock	20

In case (1) there are two ways in which the footing may fail: (1) by "punching shear," (2) by bending of the footing when column is under direct load. Consider a column of square section $d \times d$ feet and total load W. If the safe pressure on the ground be P then the area A of the footing will be W/P. If the footing be square its dimension will be $D \times D$ feet $= A$.

Referring to Fig. 19, consider the "punching shear," *i.e.* shear concentrated over a small area which tends to cause the column to punch a hole through the base. The area subject to punching shear is the perimeter of the column $\times$ depth of the footing, *i.e.* $4 \times d \times h$. The load producing the shear will be load on column — upward pressure under column, *i.e.* $W - P \times d^2$. The punching shear stress will be

$$\frac{W\left(1 - \frac{d^2}{D^2}\right)}{4h \times d}$$

If this exceeds the allowable shear stress (on plain concrete) the depth of the footing will have to be increased.

The second way in which the footing may fail is by bending as a cantilever.

Considering a strip 1 foot wide we get

$$\text{Max. B.M.} = \text{total load} \times \text{lever arm}$$

$$= P\left(\frac{D-d}{2}\right)^2 \frac{1}{2} \text{ approx.}$$

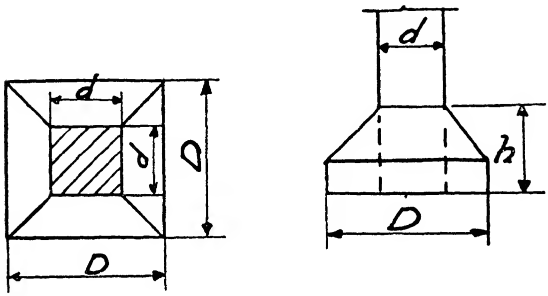


FIG. 19

Investigating the footing as a beam we find what reinforcement is required to take up the tensile stress. Note that the formulæ for rectangular bases can be found in a similar manner. The main reinforcement of the column should be taken down to 2 inches from the bottom of the footing and must be supported by small blocks of precast concrete while the footing is being poured. The ground should be "blinded" by a thin layer of concrete of a weak mix before the footing is commenced. The depth of the footing should be sufficient for the main reinforcement to develop its full compression strength by bond. If the depth is not sufficient the main bars should have their ends hooked or bent at right angles to ensure a strong joint. Concrete footings should be poured in one batch after placing of the steel reinforcement. The reinforcement should have a minimum cover of 3 inches and should be anchored at each end. It is usual to allow twice the usual shear stress for punching shear.

Example 1.—A concrete column 10 inches square having 4 - 1 inch diameter bars as main reinforcement carries a load of 100,000 lb. Design a footing on a subsoil capable of taking a pressure of $2\frac{1}{2}$ tons per square foot (1 : 2 : 4 mix).

$$\text{Area required} = \frac{100{,}000}{2{,}240 \times 2\cdot5} = 17\cdot8 \text{ sq. ft.}$$

Make footing 4 ft. 3 in. square

Consider first the shear. Perimeter of column.

$$= 4 \times 10 = 40 \text{ in.}$$

Allowable shearing stress = 2 × 75 = 150 lbs. per sq. in.

$$\text{Shearing force} = 100{,}000 - \left(10 \times 10 \times \frac{5{,}600}{144}\right)$$
$$= 100{,}000 - 3{,}900$$
$$= 96{,}100 \text{ lb.}$$

$$\text{Area required} = \frac{96{,}100}{150} = 645 \text{ sq. in.}$$

$$\text{Depth required} = \frac{645}{40} = 16\cdot12. \quad \text{Say } 16\cdot5 \text{ in.}$$

Check for bond. Value of one 1 in. dia. bar in compression

$$= 13{,}500 \times 0\cdot785 = 10{,}600 \text{ lb.}$$

Bond force developed in 16·5 in. $= 3\cdot14 \times 1 \times 16\cdot5 \times 100$

$$= 5{,}150 \text{ lbs.}$$

Now consider bending on a strip 12 in. wide.

$$\text{Max. B.M.} = 5{,}600\left(\frac{51 - 10}{2}\right)^2 \times \frac{1}{2} \times \frac{1}{12}$$
$$= 2{,}800(20\cdot5)^2 \times \tfrac{1}{12}$$
$$\therefore 97{,}000 = Q \times 12d^2 = 125\cdot7 \times 12d^2$$
$$\therefore d^2 = 64\cdot6 \quad d = 8\cdot05 \text{ in.}$$

Using an effective depth of 16·5 in., the reinforcement

$$\text{per foot width} = \frac{97{,}000}{18{,}000 \times \cdot 872 \times 16\cdot 5} = \cdot 375 \text{ in.}^2$$

$\frac{1}{2}$ in. bars at 6-in. centres give an area = ·393 in.^{2}

The overall depth of the footing could be 20 in. In order to anchor the vertical bars they should be bent at right angles and hooked at the same level as the horizontal steel. To facilitate construction the vertical bars should be lapped at just above ground or floor level.

Method where footing rests on a block of mass concrete. Pressure in tons per square foot on mass concrete should not exceed the following values:

1 : 12 mass concrete . .	5
1 : 10 " " . .	10
1 : 8 " " . .	15
1 : 6 " " . .	20

Pressure on reinforced concrete:

1 : 2 : 4 mix . . .	30
1 : 1$\frac{1}{2}$: 3 " . . .	35
1 : 1 : 2 " . . .	40

In mass concrete the mix ratio given is that of cement to total aggregate (sand plus ballast).

In the case where a column footing rests upon a block of mass (or reinforced) concrete the design is generally similar to that where the footing rests directly on the ground. This design can be adopted with advantage where the foundation has to be carried down to a considerable depth in order to obtain a good bearing.

It should be noted that the pressure tends to spread through the mass concrete block. The angle of spread may be taken as 60°. This gives a result which is on the safe side. The angle 60° should be set off as shown in Fig. 20

to obtain a minimum safe depth for the mass concrete block. Any greater depth than this is on the safe side.

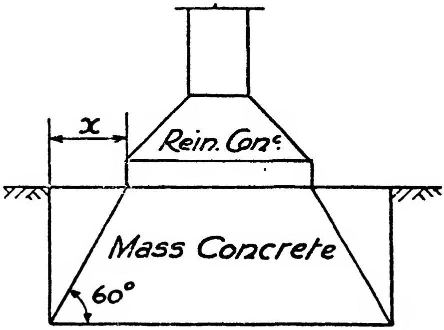


FIG. 20

In the example given the size of the mass concrete would have been 4 feet 3 inches by 4 feet 3 inches and the area of the reinforced footing for a 1 : 10 mass concrete would have been 4·45 square feet, say 2 feet 3 inches by 2 feet 3 inches. Then as the distance x in Fig. 20 would have been 1 foot the minimum depth of mass concrete would have been 1 foot 9 inches.

L.C.C. By-laws state that angle of dispersion of a load through concrete should be not less than 45° (measured from horizontal).

EXERCISES

1. A concrete column is 12 inches square (1 : $1\frac{1}{2}$: 3 mix), is 15 feet high and is hinged at each end. If the reinforcement is 4 - $1\frac{1}{8}$ diameter bars (with binders) find the safe load.

2. A concrete column 12 inches diameter is 25 feet high and has 8 - $1\frac{1}{8}$ diameter bars. Find what helical reinforce-

ment is required so that the safe load may be not less than 100,000 lb. Mix 1 : 2 : 4, column hinged each end.

3. Take column as Exercise 1 and design a footing for a bearing pressure of 2 tons per square foot.

ANSWERS

1. 152,000 lb.
2. Equivalent area of helical reinforcement 3·84 square inches.
3. 6 feet by 6 feet × 3 feet effective depth. ½ inch bars at 7½-inch centres.

Chapter VIII

REINFORCED CONCRETE CANTILEVER WALLS

Earth Pressures

IN certain cases it may be an advantage to build a retaining wall in reinforced concrete instead of a "gravity" type wall of mass concrete or similar material. The advantage is economy in the amount of concrete used. On the other hand, where a retaining wall has to be built "in trench" the cost of shuttering and reinforcement may be more than the saving in concrete, also the handling of reinforcing bars may be awkward in this case. A cantilever wall is most suitable where it can be built in the open and the earth which it has to retain filled in behind it.

Earth pressure may be calculated by means of Rankine's formula. This gives results which are rather on the safe side, but many designers prefer to use this method. The formula is intensity of pressure at any depth h below the

surface $= wh\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)$ for level surfaces. (Where w is the weight of material per cubic foot, ϕ is the angle of repose, *i.e.* the natural slope of the material).

Values of w and ϕ may be taken as given below (reproduced by permission of The Institution of Structural Engineers from Report No. 16/1943)

Material	w lb. per cub. ft.	ϕ degrees
Sand (dry)	90-100	30
„ (moist)	100-110	35
„ (wet)	110-125	25
Vegetable earth (dry)	90-100	30
„ „ (moist)	100-110	45
„ „ (wet)	110-120	15
Gravel	90	40
Rubble Stone	100-110	45
Gravel and Sand	100-110	25-30
Clay (dry)	120-140	30
„ (moist)	120-160	45
„ (wet)	120-160	15
Mud	105-120	0
Ashes	40	40

As the pressure varies with the depth, the pressure diagram will be a triangle and the total pressure will be height $\times$ average pressure

$$= h \times \frac{wh}{2}\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)$$

$$= \tfrac{1}{2}wh^2\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)$$

The total pressure will act at two-thirds h from the top.

Where the earth retained is surcharged, *i.e.* loaded by

a live load, then it is usual to allow for this by assuming an equivalent height of earth above the top of the wall. The equivalent height may be found by dividing the pressure due to surcharge by the weight of the earth per cubic foot. The pressure diagram in this case will be a trapezium and

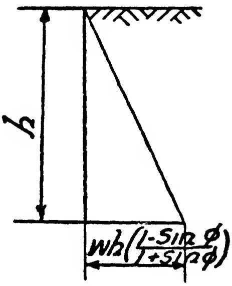


FIG. 21

not a triangle and the total pressure will not act at two-thirds *h* from the top but at a distance depending upon the amount of surcharge. When the surface is inclined the line of thrust will also be inclined.

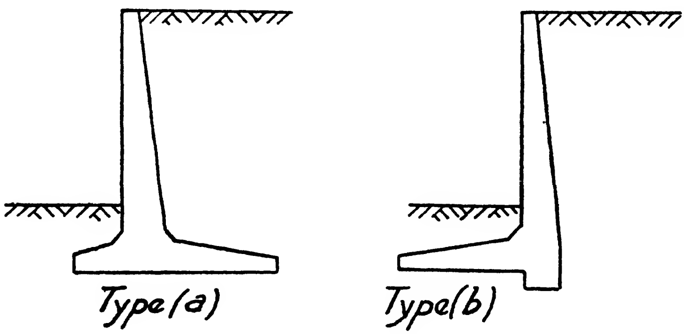


FIG. 22

For further information on the pressure on retaining walls *see* Professor Jenkin's paper on that subject and Building Research Special Report.

Cantilever walls may be either Type (*a*) or (*b*). Type (*a*) is more generally used as the weight of material retained assists in preventing the wall from overturning.

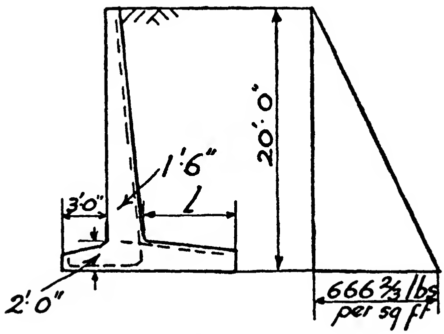


FIG. 23

DESIGN.—The vertical portion of the wall acts as a cantilever subject to bending and the amount of bending moment increases with the depth below the surface; at the same time the pressure of the earth tends to bend the base slab. There is also a tendency for the wall to move bodily forward. In Type (*b*) this is resisted by the portion projecting vertically below the base slab.

Example.—Design a retaining wall Type (*a*) to support earth having $w = 100$ lb. per cubic foot, $\varnothing = 30°$, height 20 feet and allowable pressure on ground 2 tons per square foot.

$$\text{Pressure} = wh\left(\frac{1 - \sin\varnothing}{1 + \sin\varnothing}\right)$$

$$= 100 \times h \times \tfrac{1}{3} = 33\tfrac{1}{3} \times h \text{ lb. per sq. ft.}$$

Max. pressure $= 20 \times 33\frac{1}{3}$
$= 666\frac{2}{3}$ lbs. per sq. ft.

Assume base to be 2 ft. thick.

Consider a strip of wall 1 ft. wide, then total pressure

$$= \tfrac{1}{2}wh^2\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)$$

$$= \frac{100}{2} \times \frac{1}{3} \times 18^2$$

$$= \frac{32{,}400}{6} = 5{,}400 \text{ lb.}$$

at 6 ft. above top of base.

Moment about base $= 5{,}400 \times 6$
$= 32{,}400$ ft. lbs.
$= 388{,}800$ in. lbs.

Now M.R. of section $= Qbd^2$ and $Q = 125{\cdot}7$ for 1 : 2 : 4 mix and steel stress of 18,000 lbs./in.^{2}

$$\therefore 125{\cdot}7 \times 12d^2 = 388{,}800$$
$$d^2 = 257 \text{ in.}^2$$
$$d = 16 \text{ in.}$$

B.M. at 6 ft. from top $= \frac{1}{2} \times 100 \times 36 \times \frac{1}{3} \times 24$
$= 14{,}400$ in lbs.
$\therefore d = 3$ in. approx.

B.M. at 12 ft. from top $= \frac{1}{2} \times 100 \times 144 \times \frac{1}{3} \times 48$
$= 115{,}200$ in. lbs.
$\therefore d = 8{\cdot}7$ ins.

For constructional purposes it is advisable to make the wall not less than 9 in. at top and 18 in. at bottom, then thicknesses at 6 ft. and 12 ft. from top are 12 in. and 15 in. respectively. This allows $1\frac{1}{2}$ in. min. cover to bars.

$$A_t \text{ at top of base slab} = \frac{388{,}000}{\cdot 872 \times 16 \times 18{,}800} = 1{\cdot}55 \text{ in.}^2$$

Use 1-in. diam. bars at 6-in. centres $A_t = 1{\cdot}57$ in.^{2}

$$A_t \text{ at 6-ft. from top} = \frac{14{,}400}{\cdot 872 \times 10 \times 18{,}000} = \cdot 092 \text{ in.}^2$$

1-in. diam. bars at 36-in. centres will do

$$A_t \text{ at 12 ft. from top} = \frac{115{,}200}{\cdot 872 \times 13 \times 18{,}000} = 0{\cdot}565 \text{ in.}^2$$

1-in. diam. bars at 12-in. centres give $A_t = 0{\cdot}785$ in.^{2}

The reinforcement will be "stopped off" at different levels to give required area.

BASE SLAB.—Let projection at back be l ft.

Now overturning moment $= \frac{1}{2} \times 100 \times 20^2 \times \frac{1}{3} \times 6\frac{2}{3}$

$= 44{,}444$ ft. lbs.

For stability the moment due to dead weight should be twice this. Assume l to be 8 ft.

Then dead load is as below:

		Weight (lb.)	Arm (ft.)	Moment (ft. lbs.)
Earth . .	$18 \times 8 \times 100 =$	14,400	8·5	122,000
Stem of wall	$1\frac{1}{8}$ av. $\times 18 \times 144 =$	2,916	3·56	10,400
Base of wall .	$2 \times 12{\cdot}5 \times 144 =$	3,600	6·25	24,500
		20,916		154,900

As the total moment due to dead load is more than twice the overturning moment this should be suitable. Now consider the pressure below base slab.

$$\text{Direct pressure} = \frac{20{,}916}{12{\cdot}5} = 1{,}673 \text{ lb. per sq. ft.}$$

Taking Moments about the centre of the base slab (dead loads) we get

$$\begin{array}{lrcr} \text{Earth} & 14{,}400 \times 2{\cdot}25 & = & 32{,}400 \\ & 2{,}916 \times 2{\cdot}69 & = & -7{,}244 \\ \hline & & & \underline{\underline{25{,}156}} \end{array}$$

$\therefore$ Effective Moment $= 44{,}444 - 25{,}156 = 19{,}288$ ft./lb.

Section Modulus of Base Slab $= \dfrac{1 \times 12{\cdot}5^2}{6} = 26{\cdot}04$ ft.^{3}

$\therefore$ Bending Pressure $= \dfrac{19{,}288}{26{\cdot}04} = \pm\, 740$ lb. per sq. ft.

$$\left.\begin{array}{ll} \text{Max. Pressure} = 1{,}673 + 740 = 2{,}413 \\ \text{Min.} \quad\;\, ,, \quad\;\; = 1{,}673 - 740 = \;\;\, 933 \end{array}\right\} \text{lb. per sq. ft.}$$

B.M. on projecting toe $= \dfrac{2{,}413 + 2{,}058}{2} \times 3 \times 1{\cdot}5$

$= 10{,}062$ ft. lbs.

$= 120{,}744$ in. lbs.

Reinforcement required per ft.

$$\text{run} = \frac{120{,}744}{\cdot 872 \times 20{\cdot}5 \times 18{,}000}$$

$= \cdot 374$ in.^{2}

(3-in. cover has been allowed to bars).

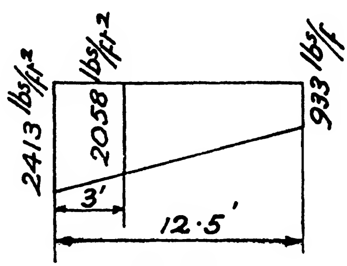


FIG. 24

1-in. bars at 18-in. centres give an area of 0·524 sq. in., therefore, if every third bar in the stem is bent down to act as slab reinforcement the arrangement will be suitable. It should be noted that in addition to the main reinforcement in the back of the wall, distribution bars should be provided in the stem and in the base slab. For this purpose $\frac{1}{2}$-in. bars at 12- to 18-in. centres will be sufficient.

The corners where the stem joins the slab should be "splayed" and "splay bars" should be inserted to take up the bending moment at these points. The bars in the stem which are not required as slab reinforcement should be anchored in the base.

SLIDING.

$$\text{Total horizontal force} = \tfrac{1}{2} \times 100 \times 20^2 \times \tfrac{1}{3}$$
$$= 6{,}667 \text{ lb.}$$
$$\text{Vertical force} = 20{,}916 \text{ lb.}$$

If coefficient of friction be 0·4, then force resisting sliding

$$= 20{,}916 \times 0{\cdot}4$$

= 8,400 lb. which is greater than 6,667 lb., therefore the wall is safe against sliding.

The tendency to sliding is increased by the presence of moisture and, therefore, weep pipe say 2- to 3-in. diam. should be provided at centres varying according to the nature of the backing. The resistance to sliding can be increased by making a slight projection on the base slab or, preferably, a slight slope on the underside of the base slab.

Chapter IX

PRESENT TENDENCIES IN R.C. DESIGN AND PRACTICE

Present tendencies in R.C. design; pre-cast concrete; pre-stressed concrete; detailing; shuttering and surface treatment; bibliography

AT the moment R.C. design is in rather a fluid condition and it is to be hoped that the Code of Practice Committee will do a great deal in clarifying various points at issue. It would appear to be a foregone conclusion that the permissible stresses will be increased in view of the improved quality of cements now available and also the improvements in workmanship and supervision. In this connection it can be noted that since the war it has been common practice to allow 10 per cent increase in working stresses. This is in line with the war-time practice in structural steelwork. For instance, for 1 : 2 : 4 mix the basic stress in bending compression becomes 750 + 10 per cent = 825 lbs./in.^{2}, with corresponding increases for direct compression, shear and bond stresses. At the same time the permissible tensile stress for mild steel becomes 18,000 + 10 per cent = 20,000 lbs./in.^{2} Other mixes will have their permissible stresses increased in the same way. The modular ratio "*m*" would seem to be a point at issue. In the Author's opinion "*m*" is a quantity which must vary with the mix, although, for any range of stress under working conditions, it can be assumed as constant for that mix.

As mentioned previously in the text, having grasped the fundamental principles of design of members such as slabs, beams and columns, the student or designer must apply these to the design of complete building frames, etc. With reference to the design of framed structures, it has been

the practice of many designers to use empirical constants for B.M. and shear. The American Society of Civil Engineers recommends that such problems be treated from first principles by application of the Hardy Cross method of moment distribution. With this method it is necessary only to assume the stiffnesses $\left(K = \frac{I}{l}\right)$ and to prepare preliminary calculations on that basis and make the final adjustments to the sections in the light of the actual stresses for the design conditions. In the design of framed structures it is inevitable that many cases are met with where a section is subject to direct compression and also bending. The eccentricity e is given by $\frac{M}{N}$ where M is the bending moment and N the direct load. Where e is small or less than half the column width the extreme fibre stress is given by

$$fc = \frac{N\left(1 + \frac{be}{D}\right)}{A_k + (m - 1)(\text{area of steel})} \qquad (D = \text{overall width})$$

In cases where e is greater than $\frac{D}{2}$ *i.e.* where equivalent load lies outside the column, the maximum stresses are found most easily by use of Mörsch's curves. Where the eccentricity is great, the member should be treated as a beam subject to combined loading.

Pre-cast concrete units have been used for some years and owing to war-time scarcity of timber for shuttering, they have been in great demand for war-time construction. It is probable that their use will continue owing to shortage of timber, especially in cases where speed in erection is essential. Many units have been standardized and particulars can be obtained from firms in this trade. Units include

beams, lintels, posts and concrete hut units, also A.R.P. shelters. Another use of pre-cast concrete is for railway sleepers either in the form of "pads" or complete sleepers. An interesting example of this class of work has been in 3-pin arch construction and the Author has recently designed an all pre-cast concrete canteen building which has been erected successfully. Spans of over 100 ft. have been designed on this principle and it is probable that quite large buildings could be erected, perhaps by use of a combination of pre-cast and *in situ* concrete. Bridges have also been renewed or built on this system and this practice seems likely to increase, especially in combination with pre-stressing. Tunnel lining segments have also been made in pre-cast concrete instead of the usual cast iron and have shown satisfactory results. It should be remembered that in pre-cast work, a good deal of the advantage of monolithic construction is lost. On the other hand, a finer concrete can be produced in the works or yard than in the field. Where vibration is applied to pre-cast concrete, a very dense and strong concrete can be produced with correspondingly higher working stresses.

Pre-stressed concrete is comparatively new in this country although it has been used successfully in France for some years by MM. Freyosinet, Lossiér, etc. No doubt the use of pre-stressing will increase as by use of this principle, lighter sections can be obtained. Bridge spans of over 100 ft. span have been produced by pre-stressing abroad and pre-stressed concrete units have been used for small bridge renewals in this country. The method of pre-stressing may vary and the operation must be carried out carefully but the principle remains the same. In flexure of R.C. beams cracks are likely to appear on or near the underside of the beams when loaded beyond a certain amount. Pre-stressing aims at removing tensile stresses from the concrete. This is done by stretching the tensile steel by means of

jacks or similar devices up to about ⅔ of the yield point stress. High yield point or moderately high yield point steel should be used for this purpose. The concrete may be poured round the reinforcement or the steel may be placed in cored holes in the concrete and grouted in. After the concrete has had time to set, the tension in the steel is released and this puts the section into the stress condition shown in Fig. 25 (*a*), *i.e.* compression in the concrete below the neutral axis and tension above. The usual stress condition is as stated in the chapter on basic theory of bending and shown in Fig. 25 (*b*). The combination of pre-stressing

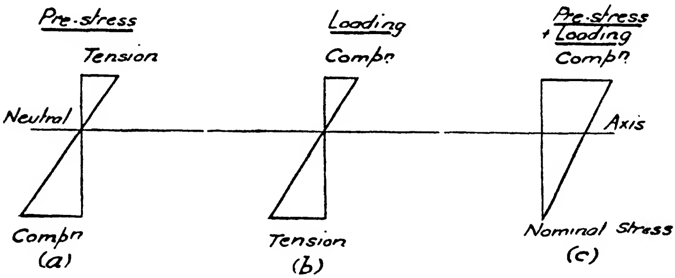


FIG. 25

and maximum loading gives the condition shown in Fig. 25 (*c*), in which the concrete is in compression throughout or very nearly so. It should be remembered that creep and shrinkage tend to reduce the effect of pre-stressing to a certain small extent. For freely supported spans pre-stressing does show economy in weight and materials, although for long spans the pre-stressing of the shear steel presents practical difficulties, and the extensive use of the principle seems to be a probable future development. Where members are subject to reversal of B.M. the application of pre-stressing becomes rather involved.

The *detailing* of reinforced concrete is very important, in fact, much of the success of any job depends on the manner in which this information is given. It is important to remember that in the past, it has often been the practice to employ unskilled labour on the site, with unfortunate results. While this may not be so prevalent to-day, it is still essential that the information be given to the men on site in such a manner that nothing is left to the imagination, with consequent loss of time and temper. Details should be kept as simple as possible in order to avoid complicated shuttering. It is better to sacrifice a little economy in material in order to simplify shuttering, as generally speaking,

BAR SCHEDULE FOR A.R.P. SHELTER

MARK	Nº OFF	DIAR.	POSITION	BENDING DETAILS	LENGTH
a	40	3/4"	FLOOR	2'-0" 11'-6" 2'-0"	16'-10"
b	40	3/4"	"	1'-0" 6" 8'-10" 6" 1'-0"	13'-2"
c	20	1/2"	"	2'-0" 11'-9"	14'-5"
d	20	1/2"	"	STRAIGHT	21'-0"
e	40	3/4"	SIDES	2'-0" 9'-0" 2'-0"	14'-4"
f	10	3/4"	"	11'-0" 2'-0"	14'-4"
g	20	1/2"	ENDS	2'-0" 8'-6" 2'-0"	13'-4"
h	10	1/2"	"	6'-0" 2'-0"	8'-10"
j	40	3/4"	ROOF	2'-0" 11'-6" 2'-0"	16'-10"
k	40	3/4"	"	1'-0" 6" 8'-10" 6" 1'-0"	13'-2"
l	20	1/2"	"	2'-0" 11'-9"	14'-5"
m	20	1/2"	"	STRAIGHT	21'-0"
n	60	3/4"	SPLAYS	3'-0"	3'-8"
o	4	"	LINTEL	4'-6"	5'-2"
p	4	"	"	6" 6" 2'-6" 6" 6"	5'-2"
q	6	"	ENTRANCE	3'-0" 2'-0"	5'-8"
r	6	"	"	STRAIGHT	6'-0"

FIG 26.

material is cheaper than skilled labour. The rules regarding concrete cover to reinforcement should be rigidly adhered to—err on the side of safety if at all. The spacing of bars is important and rules regarding this must be observed. As stated in Chapter III, the horizontal spacing should be not

less than the maximum bar diameter or ½ in. more than the maximum size of coarse aggregate (whichever is the greater) and the vertical spacing not less than ¼ in. These are minimum spacings and it should be remembered that laps or splices may have to be allowed for. It is good practice to insert short horizontal spacers between layers of bars. The maximum spacing of bars should never exceed 12 to 18 inches. Reinforcement can be supported from the shuttering during concreting. The Author has found the following method of detailing prove satisfactory (1) general arrangement drawing, (2) reinforcement details showing bars (each with an item mark, (3) bar schedule with item marks. An example of this is given in Fig. 26. Some firms supply bars bent to the particulars shown on the bar schedule which saves time on site.

SHUTTERING (or formwork or centering).—This must be designed to support the weight and pressure of the wet concrete. It must be watertight and may be of wood. Tongued and grooved boards are better than plain boards. Where a smooth finish is required, the boards should be wrought on the inside faces or lined with plywood or similar materials. Rough boards may be used where it is proposed to finish the surface by bush-hammering, rendering or plastering. Whatever the shuttering, it must be soaped or greased so that it can be struck easily without damaging the surface and used again. The formwork must be supported by horizontal joists and studding at centres from 1 ft. - 6 in. to 2 ft - 0 in. Wedges should be used in preference to nails. Metal forms may be used in cases where there is a large amount of repetition work and give satisfactory results when kept in good order. Particulars of these can be obtained from the various makers. The undersides of forms for beams should be given a slight camber to allow for the deflection due to the weight of concrete. The

following list gives the approximate time for striking shuttering after pouring (using Portland cement):

Vertical walls . . .	1 - 2 days.
Columns	2 ,,
Beam sides . . .	2 ,,
,, soffits . . .	7 ,,
Slabs and arches (soffits) .	7 ,,
Small struts . . .	7 ,,

Where rapid-hardening or high alumina cement is used, these times can be reduced. Although the cost is greater, there may be a saving where work is being done between tides or under short possession of the site.

FINISHES.—In recent years the tendency has been towards using concrete as a surface as well as a structural medium. Given proper architectural treatment, concrete can present a pleasing appearance and the weathering should not be worse than that of natural stone, provided that there is no exposure to concentrated smoke or deleterious fumes. As mentioned earlier a very fair face can be obtained by using plywood lining to the shuttering or by rubbing down the concrete. Where bush-hammering is to be used an extra ½ in. to 1 in. cover should be used as the aggregate must be exposed. Where the surface has to be rendered or plastered, the surface should be left rough in order to form a key. Coloured concrete can be produced by mixing pigments with the concrete or by external treatment. Granolithic or terrazzo finishes may be applied as finishes to floors and have good wearing properties. When considering the surface treatment it is as well to remember the question of construction and expansion joints. The expansion of concrete from heat is small and the provision of such joints is not important except in large structures. What is more important to remember is the shrinkage of concrete,

which may, if not controlled, cause unsightly cracks and rusting of reinforcement. It is best to divide the work into definite sections by means of dry joints and to make an architectural feature of these. Several bridge abutments have been treated in this way, using a moulded bead on the inside of the shuttering to give a grooved effect, the spacing of the beads being the same as the height of each "lift" of concrete. The effect is pleasing to the eye and gives the appearance of coursed stonework. The subject of architectural treatment of concrete is a very interesting one which will be of great importance with the increasing use of concrete for all types of structures.

In conclusion, the Author wishes to point out that in the space at his disposal, it has been possible to give only a brief summary of the subject of Reinforced Concrete. Such phenomena as plastic yield and creep, shrinkage, etc., have been purposely omitted and other important matters have been dealt with very briefly. For this reason, a bibliography is given for reference so that the student or designer who has learnt the essentials may pursue any particular aspect of the subject in which he is interested. The tables give the quantities of materials per cubic yard of concrete for various mixes, design factors for different values of t, c and m, and also areas of steel reinforcement, also a list of the British Standard Specifications relating to reinforced concrete.

APPENDIX

BRITISH STANDARD SPECIFICATIONS FOR REINFORCED CONCRETE

No.	TITLE
12	Portland Cement.
146	Portland Blastfurnace Cement.
915	High Alumina Cement.
410	Test Sieves.
812	Sampling and Testing of Mineral Aggregates (sands).
877	Foamed Blastfurnace Slag for Concrete Aggregates.
882	Natural Aggregates for Concrete.
1047	Coarse Blastfurnace Slag Aggregate.
15	Steel for General Building Construction.
548	High Tensile Steel for Bridges, etc.
745	Rolled Bars for Concrete Reinforcement (also Hard-drawn Steel Wire).

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TABLE IA

APPROXIMATE QUANTITIES REQUIRED PER CUBIC YARD OF CONCRETE USING SHINGLE AS COARSE AGGREGATE

Mix	Cement	Sand	Coarse Aggregate	Water
	bags	cub. ft.	cub. ft.	gallons
1 : 1 : 2 ..	7¾	9½	19	35½
1 : 1½ : 3 ..	6	11	22	32
1 : 2 : 4 ..	4⅝	11½	23	28
1 : 2½ : 5 ..	3⅞	12	24	26½
1 : 3 : 6 ..	3¼	12¼	24½	25
1 : 4 : 8 ..	2½	12½	25	22¾

TABLE IB

APPROXIMATE QUANTITIES REQUIRED PER CUBIC YARD OF CONCRETE USING BROKEN STONE AS COARSE AGGREGATE

Mix	Cement	Sand	Coarse Aggregate	Water
	bags	cub. ft.	cub. ft.	gallons
1 : 1 : 2 ..	8	10	20	37
1 : 1½ : 3 ..	6¼	11½	23	33¼
1 : 2 : 4 ..	4⅞	12	24	30
1 : 2½ : 5 ..	4	12½	25	27½
1 : 3 : 6 ..	3½	12¾	25½	26¼
1 : 4 : 8 ..	2⅝	13	26	24

Above values are based on cement weighing 90 lbs. per cub. ft. and with dry aggregate; with damp or wet aggregate the amount of water should be reduced (1 gallon of water = 10 lbs.).

1 : 2 : 4 mix for general R.C. work; 1 : 1 : 2 and 1 : 1½ : 3 mixes for high-grade R.C. work; other *mixes for mass* concrete work.

TABLE IIA

R.C. DESIGN DATA WITH $m = 15$ THROUGHOUT

Concrete Mixture	Working Stresses in lbs./in.^{2} — Steel: Tension t	Concrete: Bending c	Concrete: Shear s	Concrete: Bond s_b	Concrete: Compress'n f_c	Design Factors: t/c	n_1	a_1	Q	r
1 : 2 : 4	18,000 20,000 25,000 27,000	750	75	100	600	24·00 26·67 33·30 36·00	0·385 0·360 0·312 0·294	0·872 0·880 0·896 0·902	125·7 119·0 105·0 99·4	·008 ·00675 ·00467 ·00408
1 : 1½ : 3	18,000 20,000 25,000 27,000	850	85	110	680	21·20 23·55 29·40 31·70	0·414 0·390 0·338 0·321	0·862 0·870 0·887 0·893	151·5 143·7 127·5 121·8	·00975 ·00827 ·00575 ·00505
1 : 1 : 2	18,000 20,000 25,000 27,000	975	98	123	780	18·50 20·50 25·70 27·70	0·447 0·423 0·369 0·351	0·851 0·859 0·877 0·883	185·7 177·0 158·0 151·5	·0121 ·0103 ·0072 ·00633

TABLE IIB

R.C. DESIGN DATA WITH VARIABLE VALUE OF m

	Concrete Mixture	Modular Ratio m	Working Stresses in lbs./in.^{2} — Steel: Tension t	Concrete: Bending c	Concrete: Shear s	Concrete: Bond s_b	Concrete: Compress'n f_c	Design Factors: t/c	n_1	a_1	Q	r
ORDINARY CONCRETE	1 : 2 : 4	18	18,000 20,000 25,000 27,000	750	75	100	600	24·00 26·67 33·30 36·00	0·43 0·405 0·351 0·333	0·86 0·865 0·883 0·889	138 131·3 116 111	·009 ·0076 ·0053 ·0046
ORDINARY CONCRETE	1 : 1½ : 3	16	18,000 20,000 25,000 27,000	850	85	110	680	21·15 23·50 29·40 31·75	0·43 0·405 0·353 0·334	0·86 0·865 0·882 0·889	156 148·5 132·5 125·8	·0101 ·0086 ·0060 ·0052
ORDINARY CONCRETE	1 : 1 : 2	14	18,000 20,000 25,000 27,000	975	98	123	780	18·48 20·55 25·65 27·70	0·43 0·404 0·354 0·336	0·86 0·865 0·882 0·888	180 170 152 145	·0116 ·0098 ·0069 ·0061
d^o + 10%	1 : 2 : 4 1 : 1½ : 3 1 : 1 : 2	18 16 14	20,000	825 935 1073	83 94 108	110 121 135	660 748 858	24·25 21·35 18·60	0·425 0·429 0·429	0·858 0·857 0·857	150·5 171·5 198·0	·0088 ·0100 ·0115
HIGH GRADE	1 : 2 : 4	14	18,000 20,000 25,000 27,000	950	95	120	760	18·97 21·05 26·35 28·45	0·425 0·400 0·347 0·333	0·858 0·867 0·884 0·889	173 165 145·5 140·0	·0112 ·0095 ·0066 ·0059
HIGH GRADE	1 : 1½ : 3	12	18,000 20,000 25,000 27,000	1100	110	135	880	16·37 18·17 22·70 24·55	0·423 0·398 0·346 0·333	0·859 0·867 0·885 0·889	200·0 189·5 168·5 162·8	·0129 ·0110 ·0076 ·0068
HIGH GRADE	1 : 1 : 2	11	18,000 20,000 25,000 27,000	1250	125	150	1000	14·4 16·0 20·0 21·60	0·433 0·408 0·356 0·338	0·856 0·864 0·881 0·887	231·5 220·0 196·0 187·5	·0015 ·0013 ·0009 ·0008

For other values of m, t and c

then $\frac{t}{mc} = \frac{1}{n_1} - 1$ $\qquad a_1 = 1 - \frac{n_1}{3}$ $\qquad Q = \frac{c}{2} \times n_1 a_1$ $\qquad r = \frac{c}{2} \times \frac{n_1}{t}$

TABLE III

AREAS OF ROUND BARS (sq. in.)

(Reproduced from *Notes on Cement and Reinforced Concrete*, by permission of Ketton Portland Cement Co. Ltd.)

Dia. in.	NUMBER OF BARS										Dia. in.
	1	2	3	4	5	6	7	8	9	10	
$\frac{1}{4}$	0·049	0·098	0·147	0·196	0·245	0·294	0·343	0·392	0·441	0·491	$\frac{1}{4}$
$\frac{5}{16}$	0·076	0·153	0·230	0·306	0·383	0·460	0·536	0·613	0·690	0·767	$\frac{5}{16}$
$\frac{3}{8}$	0·110	0·220	0·331	0·441	0·552	0·662	0·772	0·883	0·993	1·104	$\frac{3}{8}$
$\frac{7}{16}$	0·150	0·300	0·450	0·601	0·751	0·901	1·052	1·202	1·352	1·503	$\frac{7}{16}$
$\frac{1}{2}$	0·196	0·392	0·588	0·785	0·981	1·177	1·374	1·570	1·766	1·963	$\frac{1}{2}$
$\frac{9}{16}$	0·248	0·497	0·745	0·994	1·242	1·491	1·739	1·988	2·236	2·485	$\frac{9}{16}$
$\frac{5}{8}$	0·306	0·613	0·920	1·227	1·534	1·840	2·147	2·454	2·761	3·068	$\frac{5}{8}$
$\frac{11}{16}$	0·371	0·742	1·113	1·484	1·856	2·227	2·598	2·969	3·340	3·712	$\frac{11}{16}$
$\frac{3}{4}$	0·441	0·883	1·325	1·767	2·209	2·650	3·092	3·534	3·976	4·418	$\frac{3}{4}$
$\frac{13}{16}$	0·518	1·037	1·555	2·074	2·592	3·111	3·629	4·148	4·665	5·185	$\frac{13}{16}$
$\frac{7}{8}$	0·601	1·202	1·803	2·405	3·006	3·607	4·209	4·810	5·411	6·013	$\frac{7}{8}$
$\frac{15}{16}$	0·690	1·380	2·070	2·761	3·451	4·141	4·832	5·522	6·212	6·903	$\frac{15}{16}$
1	0·785	1·570	2·356	3·141	3·927	4·712	5·497	6·285	7·068	7·854	1
$1\frac{1}{8}$	0·994	1·988	2·982	3·976	4·970	5·964	6·958	7·952	8·946	9·940	$1\frac{1}{8}$
$1\frac{1}{4}$	1·227	2·454	3·681	4·908	6·136	7·363	8·590	9·817	11·044	12·272	$1\frac{1}{4}$
$1\frac{3}{8}$	1·484	2·969	4·454	5·939	7·424	8·909	10·394	11·879	13·364	14·849	$1\frac{3}{8}$
$1\frac{1}{2}$	1·767	3·534	5·301	7·068	8·835	10·602	12·369	14·136	15·903	17·671	$1\frac{1}{2}$
$1\frac{5}{8}$	2·073	4·147	6·221	8·295	10·369	12·443	14·517	16·591	18·665	20·739	$1\frac{5}{8}$
$1\frac{3}{4}$	2·405	4·810	7·215	9·621	12·026	14·431	16·837	19·242	21·647	24·053	$1\frac{3}{4}$
$1\frac{7}{8}$	2·761	5·522	8·283	11·044	13·806	16·567	19·328	22·089	24·850	27·612	$1\frac{7}{8}$
2	3·141	6·283	9·424	12·566	15·708	18·849	21·991	25·132	28·274	31·416	2

TABLE IV

AREAS OF ROUND BARS IN SQUARE INCHES PER FOOT WIDTH

Dia. in.	SPACING IN INCHES																Dia. in.
	3	3½	4	4½	5	5½	6	6½	7	7½	8	8½	9	10	11	12	
$\frac{3}{16}$	0·110	0·095	0·083	0·074	0·066	0·06	0·055	0·051	0·047	0·044	0·041	0·039	0·037	0·033	0·030	0·028	$\frac{3}{16}$
$\frac{1}{4}$	0·196	0·168	0·147	0·13	0·118	0·107	0·098	0·091	0·084	0·079	0·074	0·069	0·065	0·059	0·054	0·049	$\frac{1}{4}$
$\frac{5}{16}$	0·307	0·263	0·23	0·204	0·184	0·167	0·153	0·142	0·132	0·123	0·115	0·108	0·102	0·092	0·084	0·077	$\frac{5}{16}$
$\frac{3}{8}$	0·442	0·379	0·331	0·294	0·265	0·241	0·221	0·204	0·190	0·177	0·166	0·156	0·147	0·133	0·121	0·110	$\frac{3}{8}$
$\frac{1}{2}$	0·785	0·672	0·589	0·524	0·471	0·428	0·393	0·364	0·337	0·314	0·295	0·277	0·262	0·236	0·214	0·196	$\frac{1}{2}$
$\frac{5}{8}$	1·23	1·05	0·92	0·818	0·736	0·669	0·614	0·569	0·526	0·491	0·460	0·433	0·409	0·368	0·335	0·307	$\frac{5}{8}$
$\frac{3}{4}$	1·77	1·52	1·325	1·18	1·06	0·964	0·884	0·819	0·757	0·707	0·663	0·624	0·589	0·53	0·482	0·442	$\frac{3}{4}$
$\frac{7}{8}$	2·41	2·06	1·8	1·60	1·44	1·31	1·20	1·11	1·03	0·962	0·902	0·849	0·801	0·722	0·656	0·601	$\frac{7}{8}$
1	3·14	2·69	2·36	2·09	1·89	1·71	1·57	1·45	1·35	1·26	1·18	1·11	1·05	0·943	0·857	0·785	1
$1\frac{1}{8}$	3·98	3·41	2·98	2·65	2·39	2·17	1·99	1·84	1·70	1·59	1·49	1·40	1·33	1·19	1·08	0·994	$1\frac{1}{8}$

INDEX

Aggregates, coarse, 11, 12
„ , fine, 9, 10
American, Soc. Civil Engineers, 14, 19
„ practice, 21, 37, 48, 65
Anchorage, 64, 65, 76, 87
Area, of bars, *see* Bars
„ , equivalent, 70

Bar, schedule, 92, 93
Bars, areas of, Tables III, IV
„ , cover to, 22, 23, 67, 76, 91
„ , diameter, 13
„ , spacing of, 23, 29, 58, 91, 93
Beams, doubly reinforced, 41-53
„ , ell, 36-40, 50-53
„ , rectangular, 23-30
„ , tee, 36-40, 50-53
Beaming values, sub-soils, 75
„ „ , on concrete, 78
Bending moment, 54, 61, 62
„ „ , in slabs, 31, 32
Binders, *see* Hoops
Bond, 55, 56, 65
„ , safe stresses in, 55, Tables IIA, IIB
British Standard Specifications, 7, 9, 10, 12, 13, Appendix

Cantilever walls, *see* Retaining walls
Cement, 7-9
Centering, 15, 89, 92-94
Columns, areas, 67, 68
„ , design of, 66-73
„ , foundations for, 74-80
„ , subject to bending, 66, 67, 89
Compression, reinforcement, 41-54
„ , safe stresses in, 68
„ , in steel, 41, 45, 46, 69
Concrete, mixes, 14, 15, Tables IA, IB
„ , pressures on, 78
„ , properties of, 20, 21
„ , setting time, 16
„ , strength, 20, 21
„ , tests, 17, 18
Consistency, 18, 19
Core area, 67, 72
Curing, 16
Detailing, 92
Diameter, of bars, *see* Bars
Distribution steel, *see* Reinforcement

Earth pressures, 81
Effective, depth, 25, 30
„ , length, 70, 71
Elasticity modulus, *see* Young's Modulus
Ell beams, *see* Beams
Equivalent section, *see* Area, equivalent
Finishes, *see* Surface treatment
Fabric, *see* Mesh
Foundations, footings, *see* Columns
French Government formula, 31
Frost, precautions against, 18

Hard-drawn steel wire, *see* Steel
Hardy Cross method, 89, 97
Helical reinforcement, 67, 68, 69, 72, 74
Hooks, sizes of, 65
Hoops, diameter of, 13, 74
„ , spacings for, 57, 58, 68

Inertia, moment of, 70
Inst. of Structural Engineers, Reports, 81, 97

Laps, in bars, 23, 74, 78
Lever arm, 26, 38, 41, 44, 47, Tables IIA, IIB
L.C.C. By-laws, 12, 23, 70, 79

Materials, 7-13
Mesh, 13
Mixing, 14, 15
Modular ratio, 21, 22, 24, 88
Modulus, hydraulic, 8
„ , silica, 8
„ , section, 86
Mörsch's curves, 89
Moulds, *see* Centering

Neutral axis, 24, 26, 38-40, 44, Tables IIA, IIB
Percentage, of reinforcement, 26-28, 68, Tables IIA, IIB
Pitch, of stirrups, 58
„ , of hoops, 68
Placing, of concrete, 15, 16, 90
Pre-cast units, 89, 90
Pre-stressing, 90, 91
Punching shear, 20, 75, 76

Rankine, formula, 80, 81
„ , Grashof, and 33

Index

Reinforcement, *see* Bars
„ , diagonal, 59, 60
„ , double, 41-54
„ , in columns, 68, 69
„ , percentage, 26, 68
„ , secondary, 29, 32, 68, 87
„ , shear, 57-65
„ , single, 24, 25
Repose, angle of, 81
Resistance, moment of, 26, 46
Retaining walls, 80-87

Safety factor, 20
Sand, *see* Aggregates, fine
Shear, force, 56
„ , punching, 20, 75, 76
„ , reinforcement, 57-65
„ , stress, 20, 21, 40, 60
Shingle, *see* Aggregates, coarse
Shuttering, *see* Centering
Slabs, design of, 29-34
„ , doubly reinforced, 31-34
„ , singly reinforced, 29-30
Slump test, 18, 19
Spiral reinforcement, *see* Helical reinforcement
Splices, *see* Laps
Steel, mild, 13, 21, 69
„ , high tensile, 13, 21, 69
„ , hard drawn wire, 13, 28
„ , ratio, 26, Tables IIA, IIB
„ beam theory, 52, 58
Stirrups, *see* Hoops
Sub-soils, pressures on, 75

Tee beams, *see* Beams
Temperature, effects of, 16
„ , rise in, 16

Vibration, 15, 16, 90

Water, 12
Water-cement ratio, 16, 17
Working stresses, 22, 23, 48, 58, 68, 88, Tables IIA, IIB

Young's Modulus, 15, 16, 90